Uncertainty and Volatility in Financial Markets

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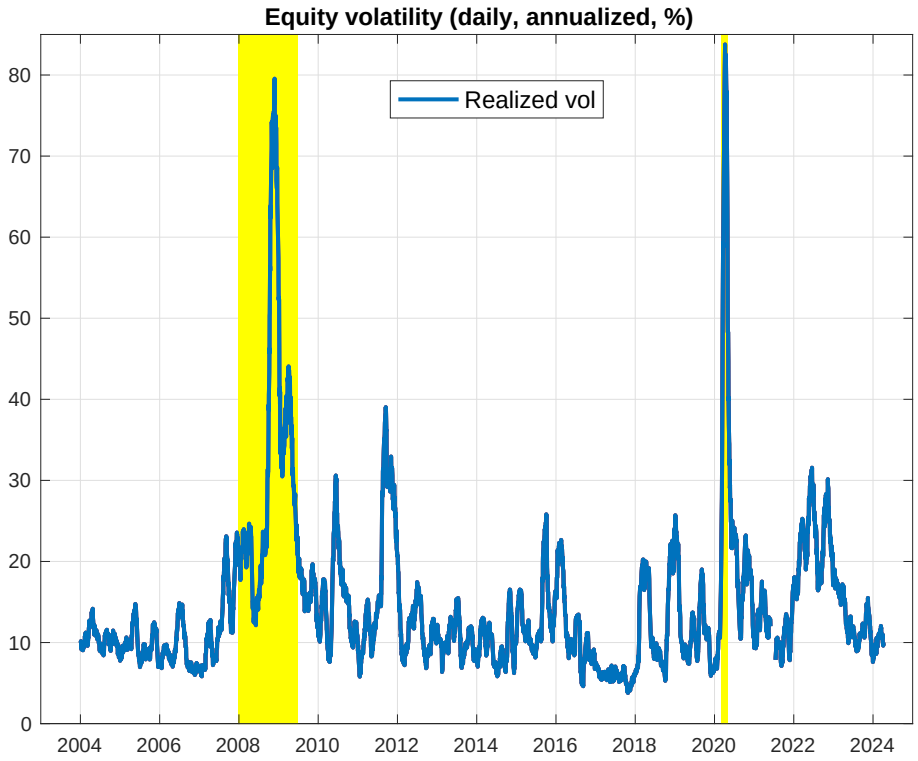
Volatility—What is it?

Volatility—How do we measure it?: I



Bird's eye view

- Why should we care about volatility?
  - High volatility may contribute to undermine our confidence in the economy
- Where does it come from?
  - Do macroeconomic developments lead to volatility or does volatility contribute to macro developments? ↷
  - Reflexivity and endogenous risk—self-reinforcing mechanisms
  - Market microstructure?
- Uncertainty?

Uncertainty and volatility

- Distinction between risk and uncertainty
  - *Risk*: we know how to determine the probabilities of occurrence of future events (Gauss, 1809)
  - *Uncertainty*: we don't even know these probabilities (Keynes, 1921; Knigth, 1921)

- Uncertainty is very bad
  - economic agents act according to worst-case scenarios
  - the economy isn't given the best opportunities to grow

Remedies?

- None (obvious), really
- Can we *mitigate* uncertainty?—Well, we can "freeze" *volatility*
- How?
  - ▷ rely on financial instruments that pay off in worst-case scenarios—e.g., out-of-the money options
  - ▷ the price of a certain basket of these instruments is the "price of volatility"
- Which price, and which instruments?
  - – This talk

Main ideas behind these innovations

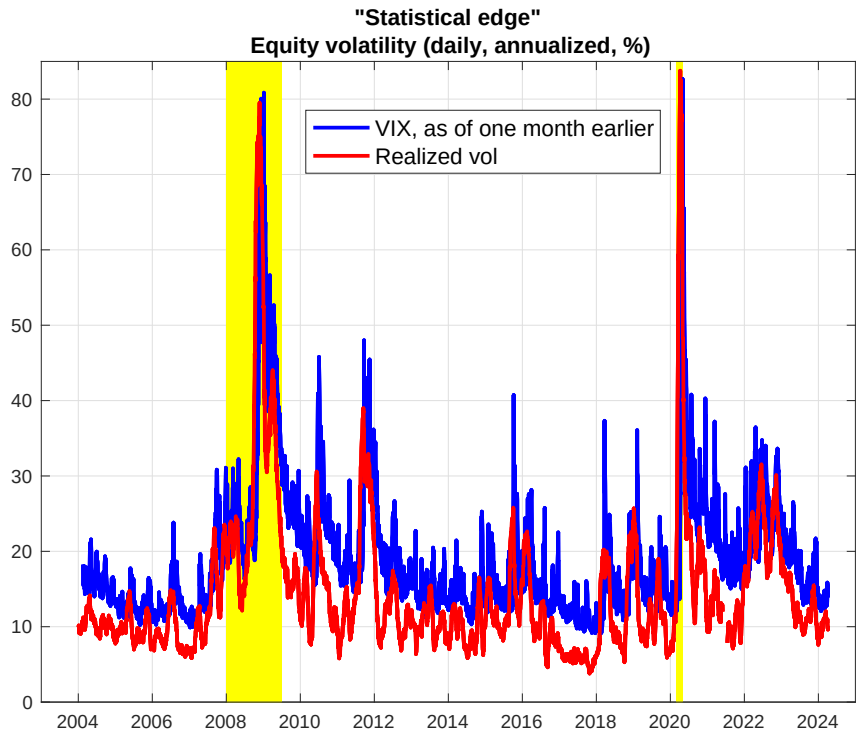
- To lock-in future volatility through dedicated contracts

- By going through such contracts, you are guaranteed the volatility you'll experience is equal to some quantity known in advance
  - Mitigating uncertainty?—we're killing the "volatility of volatility" really
  - Well, actually, you may insure against any level of volatility—You may even kill volatility

- Volatility links to uncertainty and I'll try to explore these linkages. Things are quite intricate:
  - No uncertainty doesn't mean no volatility
  - Does mitigating volatility help mitigate uncertainty?—Not necessarily
  - But we can mitigate volatility with instruments that help mitigate uncertainty

Volatility—How do we measure it?: II

Expected volatility corrected for risk



Uncertainty and volatility again

- What's remarkable is that the mathematics underlying these contracts involve certain averages of prices of **out-of-the-money** derivatives
  - out-of-the money derivatives pay off over worst-case scenario events
- Economic theory suggests that aversion to uncertainty leads us to take decisions based on worst-case scenarios

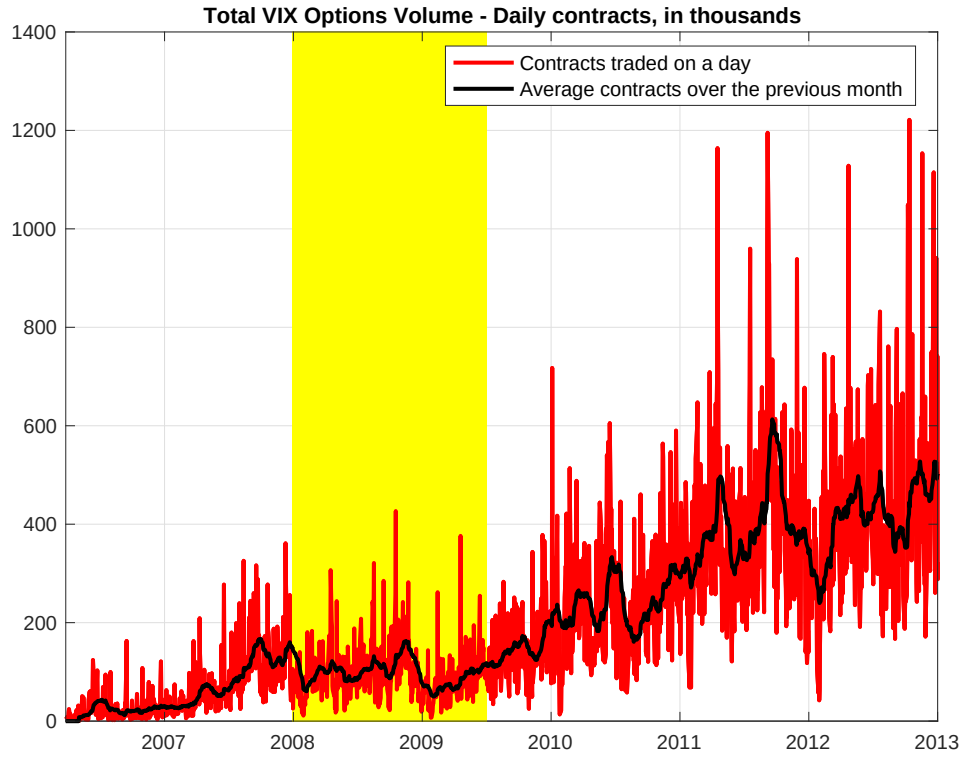
Financial innovation

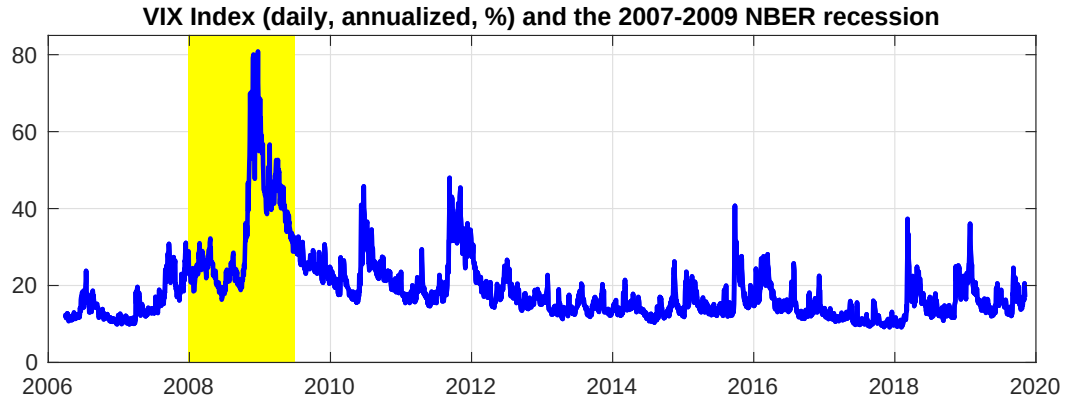
$\Longrightarrow$ The process of creation of new securities that have the potential to fill in some initial "incomplete" market structure
(Goes w/o saying: need regulation + supervision to avoid unintended *boomerang effects*.)

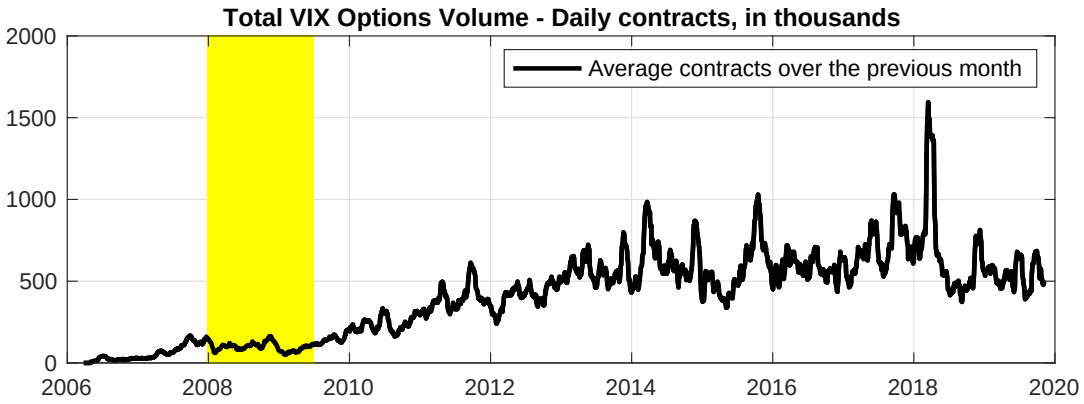
- There are events financial intermediaries seek for protection, e.g.,
  - ✓ a sudden deterioration of liquidity conditions in decentralized, sometimes opaque, interest rate markets
  - ✓ high exposure of fund managers to capital market volatility—private bankers are exposed to such a risk in a globalized and highly competitive landscape struggling for returns

- All this isn't merely academic purism ↷

First innovation wave: equity volatility (1990s-2000s)




VIX Index (daily, annualized, %) and the 2007-2009 NBER recession
80
60
40
20
0
2006
2008
2010
2012
2014
2016
2018
2020


Total VIX Options Volume - Daily contracts, in thousands
Average contracts over the previous month
2000
1500
1000
500
0
2006
2008
2010
2012
2014
2016
2018
2020

Which volatility?

- Second innovation wave: fixed income volatility (2010s-2020s)
- Why *Fixed income/interest rate volatility*?
  - In over-the-counter markets: $\sim \$500$ trillion notional regarding interest rate derivatives
  - On exchanges: $\sim \$35$ trillion futures and $\sim \$60$ trillion option contracts in the interest rate space

$$Derivatives\ (hence,\ volatility) = Fixed\ Income$$

- Hedging interest rate risk could not be more timely during periods of heightened monetary policy uncertainties

Fixed income volatility?

- Can be twice as high as Nasdaq volatility
- Complexity:
  - Fixed income markets are characterized by quoting conventions and a level of complexity that make it difficult to price volatility
- New indices of interest rate volatility maintained by the "home" to volatility trading—Chicago Board Options Exchange
  - Potential to become new "popular" gauges
  - Benefits for investors & insights for monetary policy makers

Before we start—Caveat: is low volatility good?

"No doubt, certainty is what drives one insane"

[Friedrich Nietzsche]

- High volatility may induce us to act as if we were living in a world of uncertainty, not risk
  - Volatility-uncertainty spirals (Mele & Sangiorgi, in progress)
  - Certainly society does not benefit from high volatility
- But is low volatility good?—A paradox ↷
  - Volatility is like cholesterol—there is good and bad volatility

Outline of the remaining of this talk

1. Measuring volatility in capital markets
   - Locking volatility
   - Market volatility gauges: formulation and historical perspective
   - Volatility as an asset class

2. Learning from history
   - Interest rate volatility over the centuries
   - The cyclical behavior of market volatility
   - Volatility and macroeconomic developments
   - Volatility paradox
   - Reacting to uncertainty shocks?
   - Some critical episodes
     * Lehman Brothers | Covid-19 | Inflation + Russia-Ukraine conflict

1. Measuring volatility in capital markets

Locking volatility

- Suppose you suffer a loss from some stock price or interest rate volatility arising within a month
  - Denote this volatility with $\tilde{V}$ ("realized volatility")
  - Your utility within a month is $-\tilde{V}$—this is obviously random
- You may go for the following contract
  - Pay K today
  - Receive an amount of \$ equal to $\Pi = \tilde{V} - \bar{V}$ within a month—the constant $\bar{V}$ is agreed today
  - Your utility within a month is frozen at $-RV + \Pi = -\bar{V}$—the contract may even specify $\bar{V} = 0$
  - Of course, you need to pay K today

- We have (fair value)

$$K = e^{-r(T-t)}(\mathbb{E}_t(\tilde{V}) - \bar{V}),$$

  where $T - t$ is $\frac{1}{12}$ and r is the safe rate—to simplify, a constant—and $\mathbb{E}_t$ is the expectation taken under the risk neutral probability

- This contract covers different possibilities
  - if $K = 0$, contract is a variance swap
  - if $\bar{V} = 0$, contract is a variance forward

- Financial theory provides us with expressions for $\mathbb{E}_t(\tilde{V})$ and these expressions are "model-free"

- Quick overview: VIX formulae

- *Price volatility index* for a time span $[t, T]$,[1]

$$\mathrm{VIX}_t = \sqrt{\frac{1}{T-t}\frac{2}{\mathcal{N}_t}\left(\int_0^{F_t}\frac{\mathrm{Put}_t\left(K,T\right)}{K^2}dK + \int_{F_t}^{\infty}\frac{\mathrm{Call}_t\left(K,T\right)}{K^2}dK\right)},$$

  where

  - F_t is the price of a forward contract (e.g., on equity)
  - $\mathrm{Put}_t\left(K,T\right)$ and $\mathrm{Call}_t\left(K,T\right)$ are prices of **out-of-the-money** puts and calls on the forward, expiring at T and struck at K
  - $\mathcal{N}_t$ is the market numéraire

[1] Demeterfi, K., E. Derman, M. Kamal, and J. Zou (1999). "More Than You Ever Wanted To Know About Volatility Swaps." *New York: Goldman Sachs, Quantitative Strategies Research Notes*, March.

- *Gaussian volatility* (relevant in the fixed income space)[2]

$$\text{IR-VIX}_t = \sqrt{\frac{1}{T-t}\frac{2}{\mathcal{N}_t}\left(\int_0^{R_t^F} \text{Put}_t\left(K,T\right)dK + \int_{R_t^F}^{\infty} \text{Call}_t\left(K,T\right)dK\right)},$$

where

– R_t^F is a forward swap rate, a CDS spread etc.
– ...

[2]Mele, Antonio & Yoshiki Obayashi (2011): *United States Patent No.: US 8,510,210 B1*: "Methods and Systems for Creating an Interest Rate Swap Volatility Index and Trading Derivative Products Based Thereon." Priority date: June 20, 2011.

- Model-free
  - Modeling assumptions do not show up in the final formulae
  - We basically only need the price of traded securities
  - We can build up an ecosystem around these framework

- For an overview + general methodology
  - Mele, Antonio and Yoshiki Obayashi (2015): *The Price of Fixed Income Market Volatility.* Springer Verlag, Springer Finance Series, New York

- Note that VIX_t and $\mathrm{IR\text{-}VIX}_t$ are averages of out-of-the money options in two different spaces relating to prices or interest rates
  - these are derivatives you purchase to protect from the realization of worst-case scenario events
- There must be a link between the previous formulae (the price of volatility) and aversion to uncertainty
  - We still lack theories regarding this link

Market volatility gauges: formulation and historical perspective

- Which volatility?
  - Equity
  - Fixed income + Credit
- Which concept of volatility we are trying to measure?
  - Forward looking indicators of market volatility
- Methodology
  - Skipped—(for completeness, provide some formulae below)
  - Please see previous references

- Forward looking gauges of market volatility in the fixed income space
  - Mele & Obayashi + Cboe
- As for the equity VIX, the gauges of volatility that we've introduced aggregate derivative prices related to the realization of extreme events
  - It's a new asset space, really—goes beyond equity volatility
  - As (perhaps even more than) with equity VIX, comes out from sophisticated trading activities

- Aggregate out-of-the money options (just as in the equity VIX case)
  - ▷ Expected volatility (under the risk-adjusted probability) in the time interval $[t, T]$ (today is t)
    - ▶ Can be interpreted as the fair value of a variance swap in some appropriate risk space
    - ▶ In some cases, it's "model free"

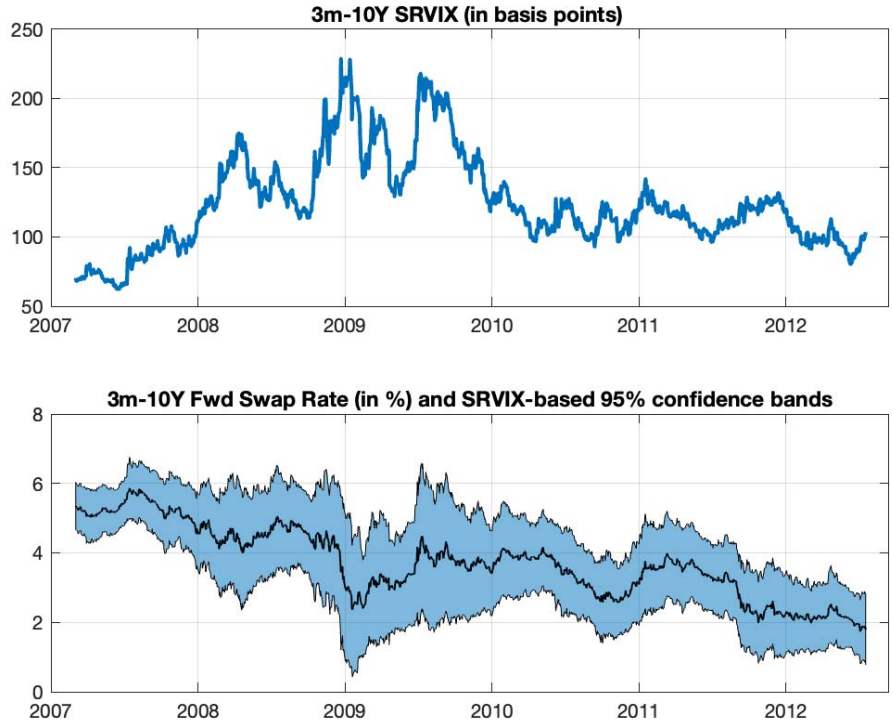
- Fear gauges (just as in the equity VIX case)

- ▶ *Main lesson*: volatility does not really seem to bring too much information

  But when it rains, it pours! ↷

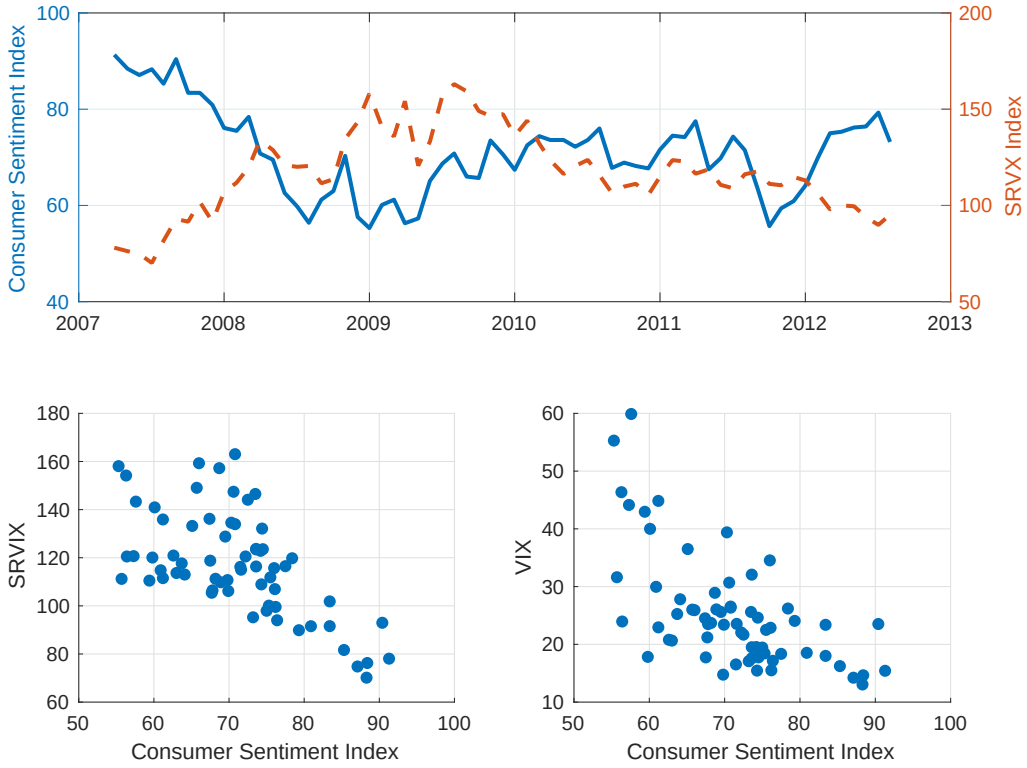
Equity volatility: VIX

*Interest rate volatility: Swaps—Cboe** (First volatility index expressed in basis points) - launched in 2012

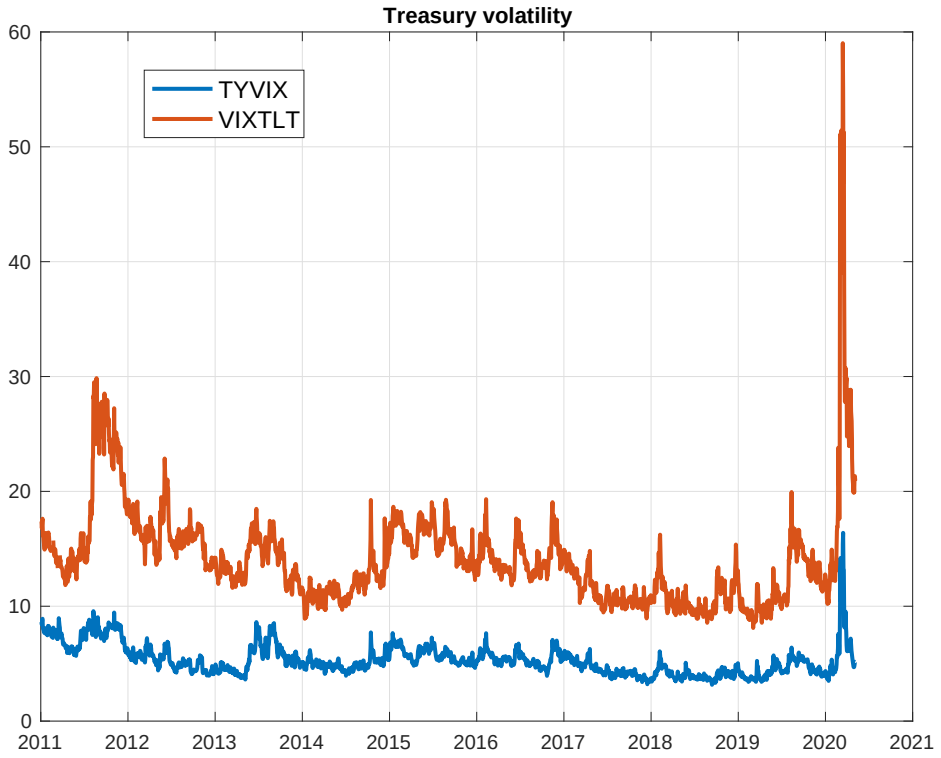


*Discontinued in 2022—now with Parameta-ICAP

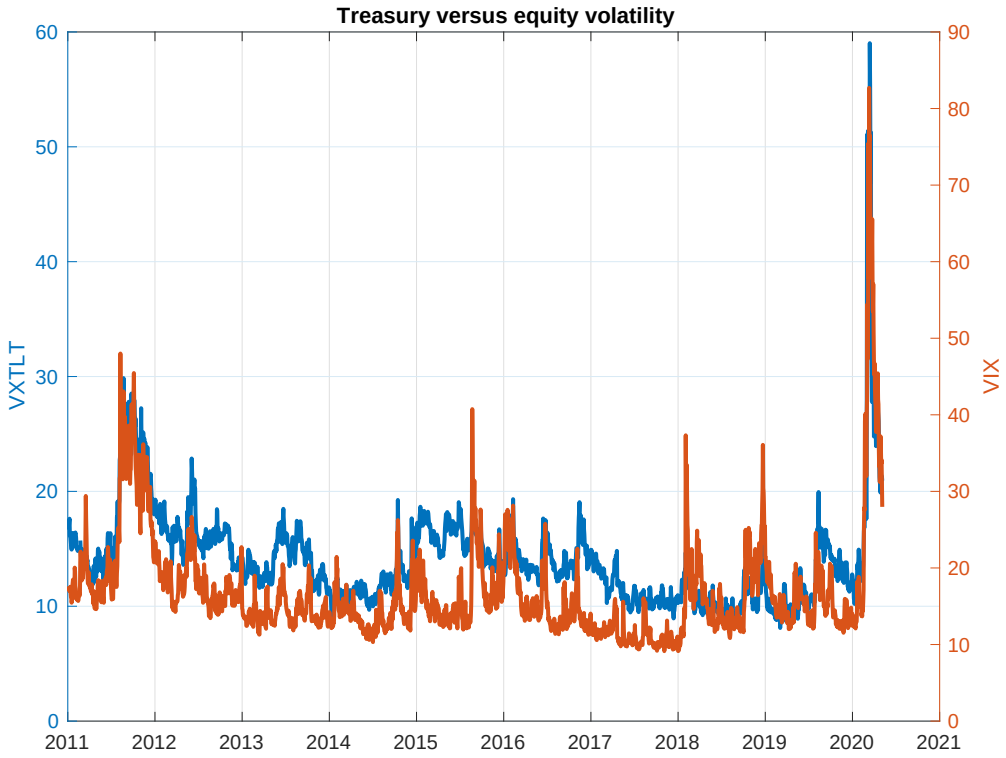
Connections to the real economy—Sentiment



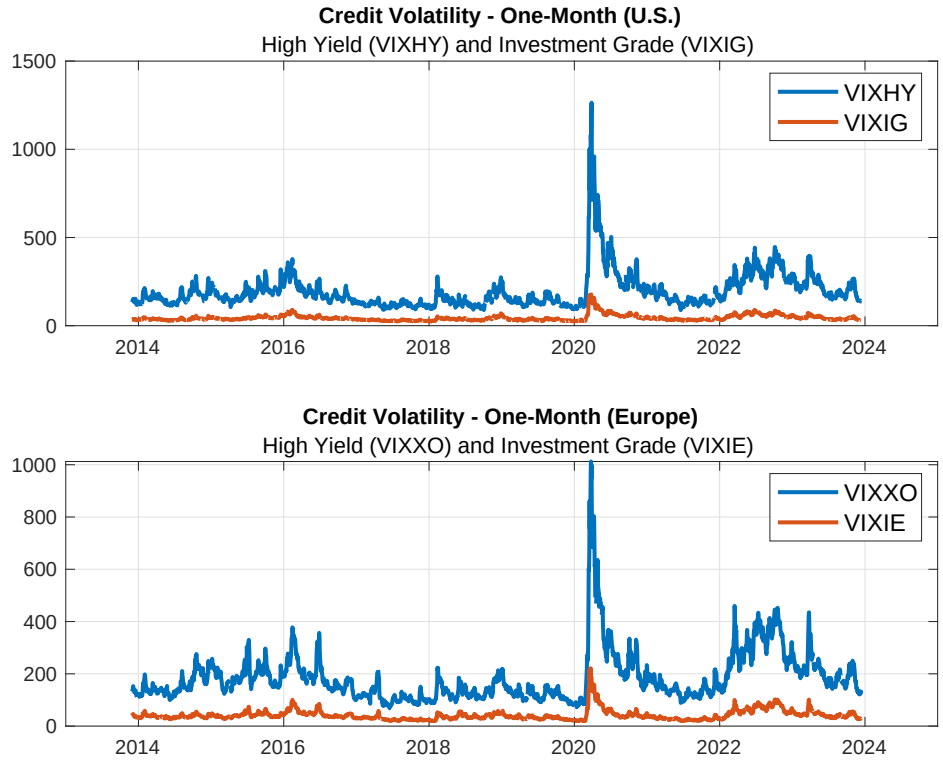
Interest rate volatility: Treasury—Cboe - launched in 2013



Treasury vs equity volatility



Credit volatility—Cboe + S&P - launched in 2023



- *Price volatility index* (Equity space—Demeterfi, Derman, Kamal, Kani, 1999). Under assumptions,[3]

$$\begin{aligned}&\mathrm{VIX}_t\left(T\right)\\ =\ &\sqrt{\frac{1}{T-t}\frac{2}{e^{-r(T-t)}}\left(\int_0^{F_t(T)}\frac{\mathrm{Put}_t\left(K,T\right)}{K^2}dK+\int_{F_t(T)}^{\infty}\frac{\mathrm{Call}_t\left(K,T\right)}{K^2}dK\right)},\end{aligned}$$

  where

  - $F_t\left(T\right)$ is the price of a future (for delivery at T) on S&P
  - $\mathrm{Put}_t\left(K,T\right)$ and $\mathrm{Call}_t\left(K,T\right)$ are prices of out-of-the-money puts and calls on the future, expiring at T and struck at K
  - Finally, r is the (constant) short-term rate (numéraire is the inverse of money market account, $\mathcal{N}_t=e^{-r(T-t)}$)

[3] Formulae provided below must be adjusted to account for the discreteness of available derivatives data.

- *Price volatility index* (Treasury space, Mele & Obayashi, 2013). Under assumptions,

$$\begin{aligned}
&\text{T-VIX}_t\left(T,\mathbb{T}\right)\\
=\;&\sqrt{\frac{1}{T-t}\frac{2}{\mathcal{N}_t}\left(\int_0^{F_t(T,\mathbb{T})}\frac{\text{Put}_t\left(K,T,\mathbb{T}\right)}{K^2}dK+\int_{F_t(T,\mathbb{T})}^{\infty}\frac{\text{Call}_t\left(K,T,\mathbb{T}\right)}{K^2}dK\right)},
\end{aligned}$$

  where

  - $F_t\left(T,\mathbb{T}\right)$ is the price of a forward (for delivery at T) on a coupon-bearing Treasury maturing at time $\mathbb{T}$
  - $\text{Put}_t\left(K,T,\mathbb{T}\right)$ and $\text{Call}_t\left(K,T,\mathbb{T}\right)$ are prices of out-of-the-money puts and calls on the forward, expiring at T and struck at K
  - The market numéraire $\mathcal{N}_t = P_t\left(T\right)$, the price of a zero coupon bond expiring at time T—this is not trivial

- *Interest rate volatility* (Swap space, Mele & Obayashi, 2011) Under assumptions,

$$\begin{aligned}&\text{IR-VIX}_t\left(t,T,\mathbb{T}_n\right)\\ =\ &\sqrt{\frac{1}{T-t}\frac{2}{\mathcal{N}_t}\left(\int_0^{R_t^F(\mathbb{T}_n)}\text{Swpn}_t^{\text{R}}\left(K,T;\mathbb{T}_n\right)dK+\int_{R_t^F(\mathbb{T}_n)}^{\infty}\text{Swpn}_t^{\text{P}}\left(K,T;\mathbb{T}_n\right)dK\right)},\end{aligned}$$

  where

  - $R_t^F\left(\mathbb{T}_n\right)$ is the forward swap rate for a tenor (extending up to some time) $\mathbb{T}_n$
  - $\text{Swpn}_t^{\text{R}}\left(K,T;\mathbb{T}_n\right)$ and $\text{Swpn}_t^{\text{P}}\left(K,T;\mathbb{T}_n\right)$ are the prices of out-of-the-money swaptions receiver and payers, expiring at T, struck at K, and with tenor $\mathbb{T}_n$
  - The market numéraire $\mathcal{N}_t$ is the Price Value of Basis Point

- *Credit volatility* (created out of options on credit default swap indices, Mele & Obayashi, 2011 and 2015) Under assumptions,

$$\begin{aligned}&\text{C-VIX}_t\left(t,T,M\right)\\ =\ &\sqrt{\frac{1}{T-t}\frac{2}{\mathcal{N}_t}\left(\int_0^{\text{CDX}_t^F(M)}\text{Sw}_t^{\text{R}}\left(K,T;M\right)dK+\int_{\text{CDX}_t^F(M)}^{\infty}\text{Sw}_t^{\text{P}}\left(K,T;M\right)dK\right)},\end{aligned}$$

where

  - $\text{CDX}_t^F\left(M\right)$ is the forward CDS index spread and M is the index's time to maturity
  - $\text{Sw}_t^{\text{R}}\left(K,T;M\right)$ and $\text{Sw}_t^{\text{P}}\left(K,T;M\right)$ are the prices of out-of-the-money option receiver and payers on the index, expiring at T, struck at K, and with tenor $\mathbb{T}_n$
  - The market numéraire $\mathcal{N}_t$ is the *defaultable* Price Value of Basis Point

Volatility as an asset class

You can't hegde your risks by trading an index in our imperfect world

- But you can trade listed products on the index such as futures and options!
  - Black swan trading—VIX call options pay off once volatility is higher than a strike
  - Volatility of volatility
- Market makers pump up liquidity into derivatives markets whilst they intermediate (and hedge) VIX futures & options over a virtuous feedback loop
- Pricing of derivatives on interest rate volatility: Mele, Obayashi, Yang (2020)

2. Learning from history

Interest rate volatility over the centuries

A signature of secular stagnation: the secular decline in nominal and real interest rates

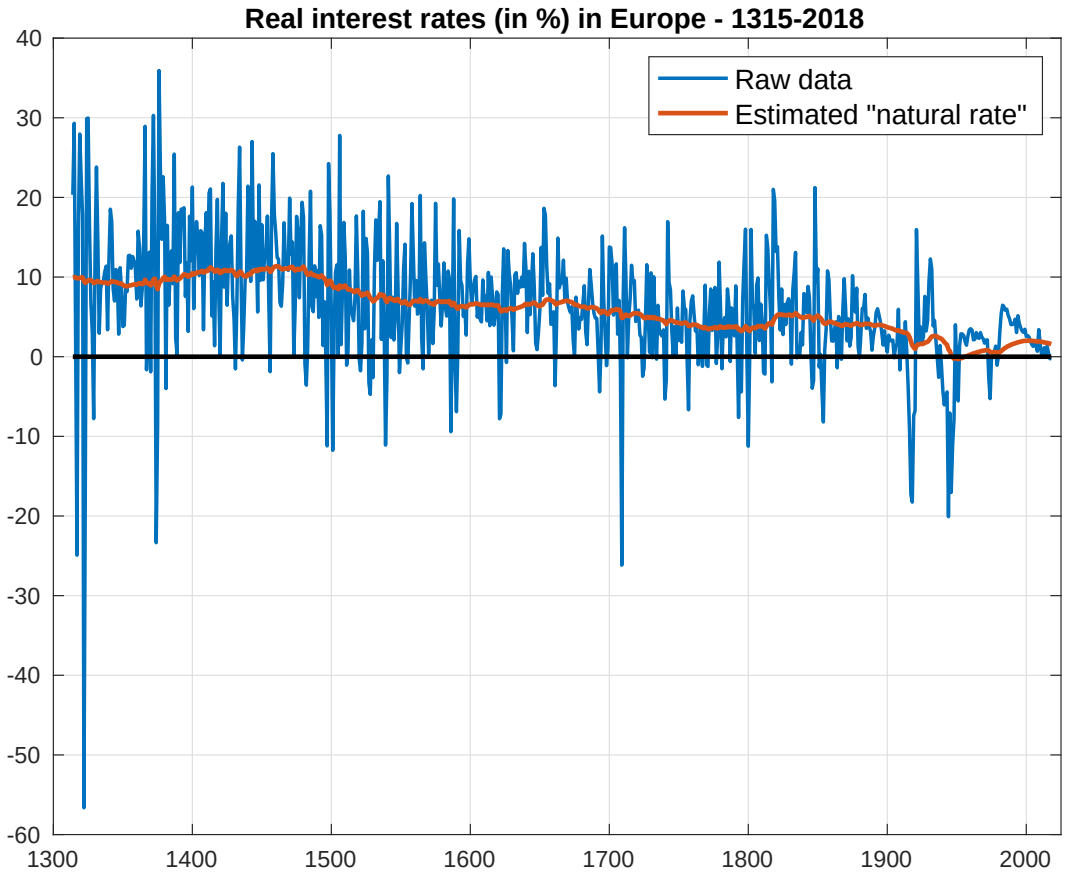
- Natural interest rate (real): the interest rate that leads to full output

$\Longrightarrow$ Sometimes a.k.a. "R-star"
  - Related to Wicksellian interest rate (the rate at which inflation is stable)

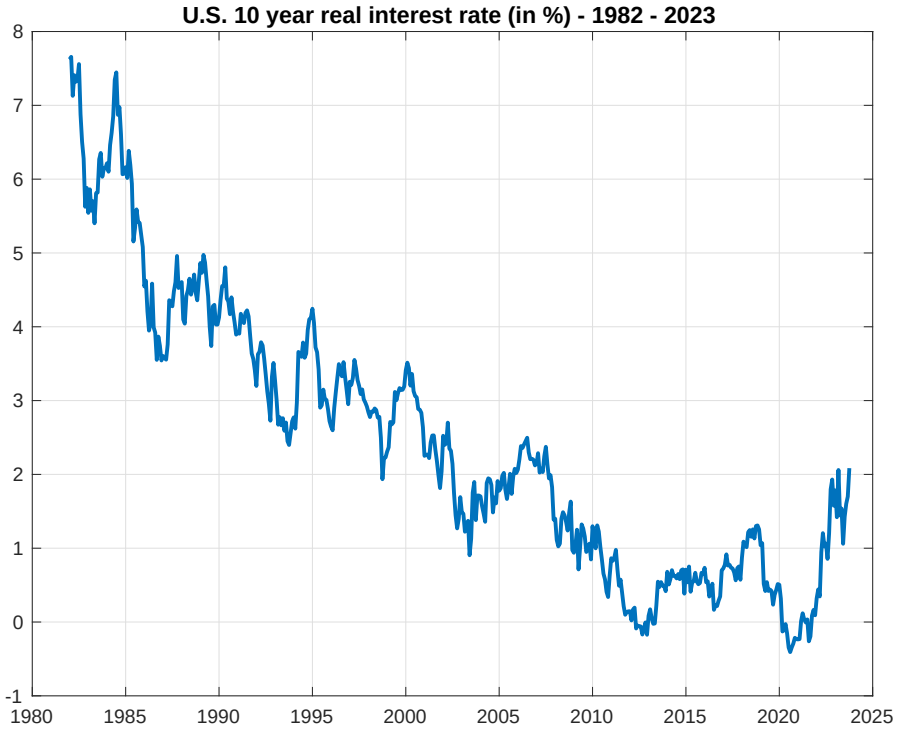
- Difficult to reconstruct

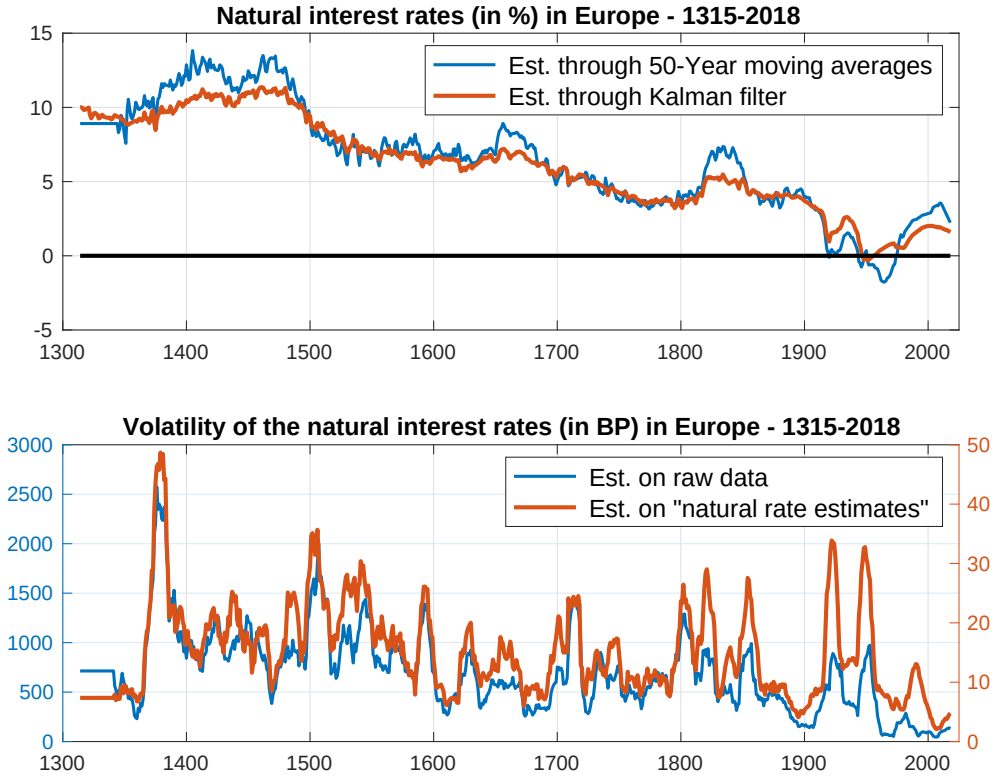
- I use Kalman filter to estimate it—similar to Jordà, Singh and Taylor (2020)

- Then, I estimate volatility on it


Real interest rates (in %) in Europe - 1315-2018
Raw data
Estimated "natural rate"
40
30
20
10
0
-10
-20
-30
-40
-50
-60
1300
1400
1500
1600
1700
1800
1900
2000

Estimates of U.S. 10 year real interest rate—Cleveland Fed 10-year real interest rate

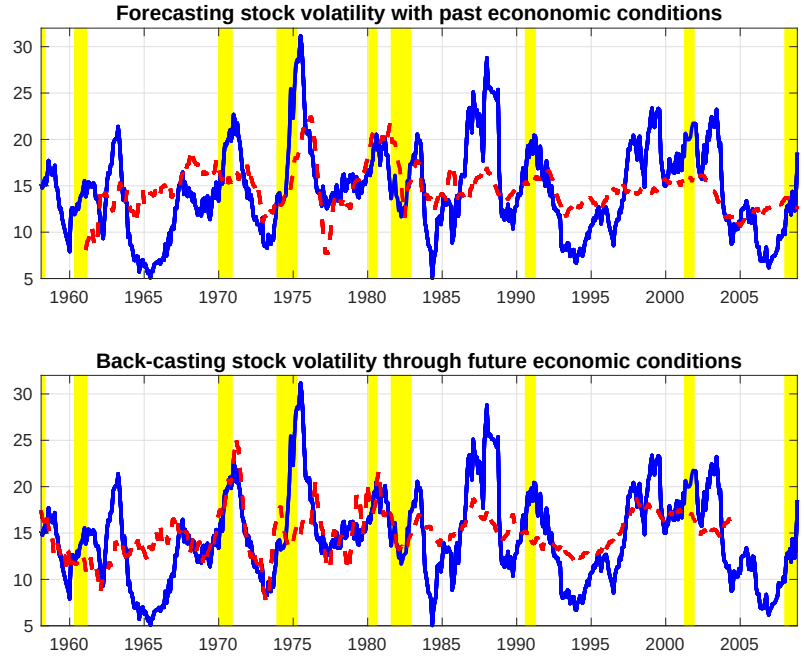




* Volatility estimates are obtained through rolling windows of ten years of data

The cyclical behavior of market volatility

Why?—Mele, 2017, 2022 [Source: Mele, 2022]



Volatility and macroeconomic developments

- *Post hoc ergo propter hoc?*

- May have policy implications

  ✓ Can you affect the economy by acting on volatility?
  ✓ Or does volatility merely reflect macroeconomic developments?
  $\Longrightarrow$ *Laissez-faire!*

- Leading or coincident indicator?—Fornari and Mele; 2013; Mele, 2022 [Source: Mele, 2022]

Consider the following regression

$$\sigma_t = c + \sum\nolimits_{i\in\{3,12,24,36\}} b_i \sigma_{t-i} + c_1 \mathbb{I}_{t\in\mathcal{O}(\mathrm{NBER}_t=1)} + c_2 \mathbb{I}_{\mathrm{NBER}_t=1} + \epsilon_t,$$

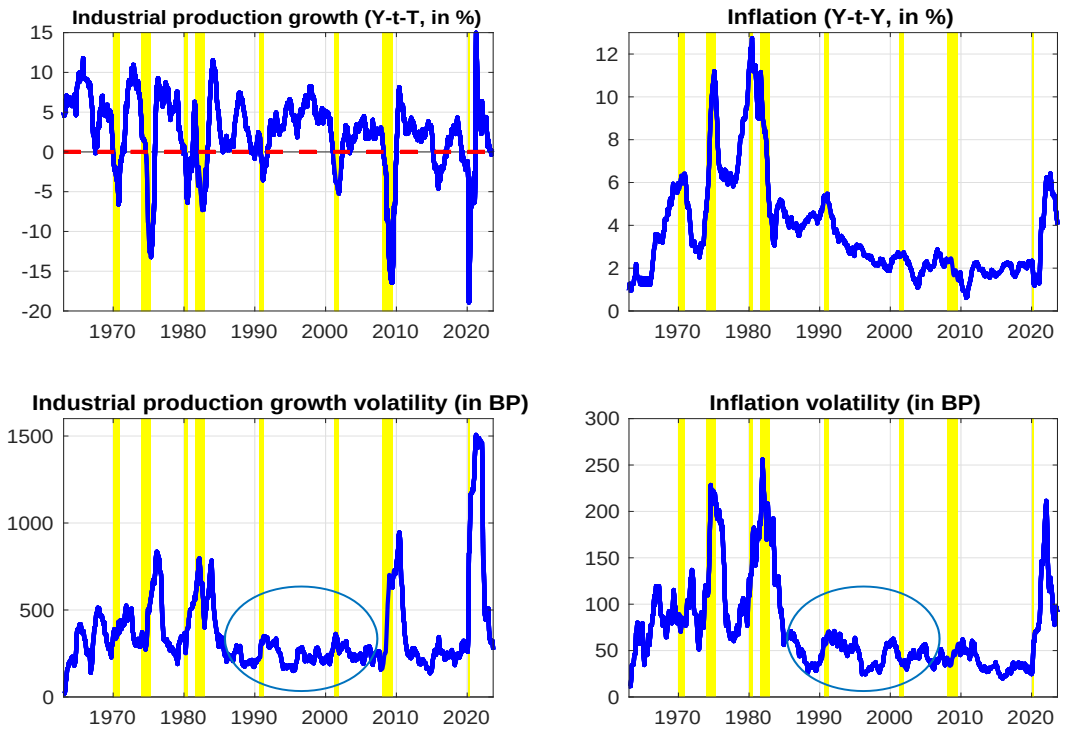
	c	b_3	b_{12}	b_{24}	b_{36}	c_1	c_2
1957-2008	3.11	0.94	-0.15	-0.01^*	-0.01^*	0.48	1.51
1957-1982	3.60	0.98	-0.24	0.02^*	-0.04	0.34^*	1.87
1983-2008	2.88	0.94	-0.09	-0.05^*	-0.01^*	1.01	1.22

Post-WWII data + two sub-samples (i) prior and over (ii) the Great Moderation. Starred figures are not statistically distinguishable from zero at the 95% level.

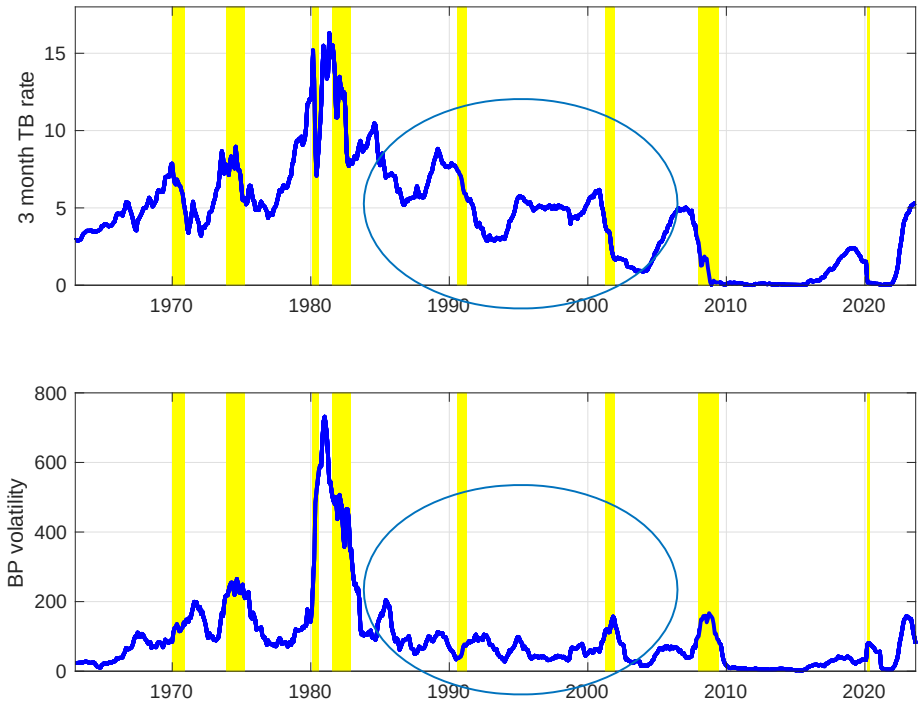
Volatility paradox

- Volatility as cholesterol: good and bad
- Volatility paradox: too little volatility may lead agents to take on excessive risky behavior
  - Brunnermeier & Sannikov (2014): risk may build up precisely in times with low volatility
  - Mele (2021): governments' debt accumulation is higher in economies with lower volatility

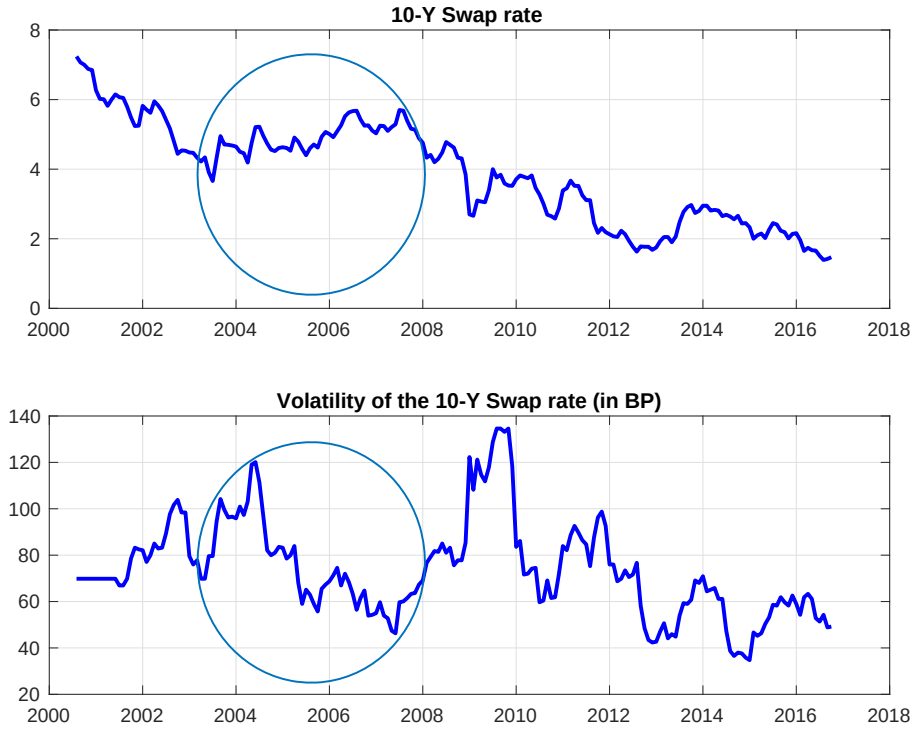
Great Moderation: 1982-2008



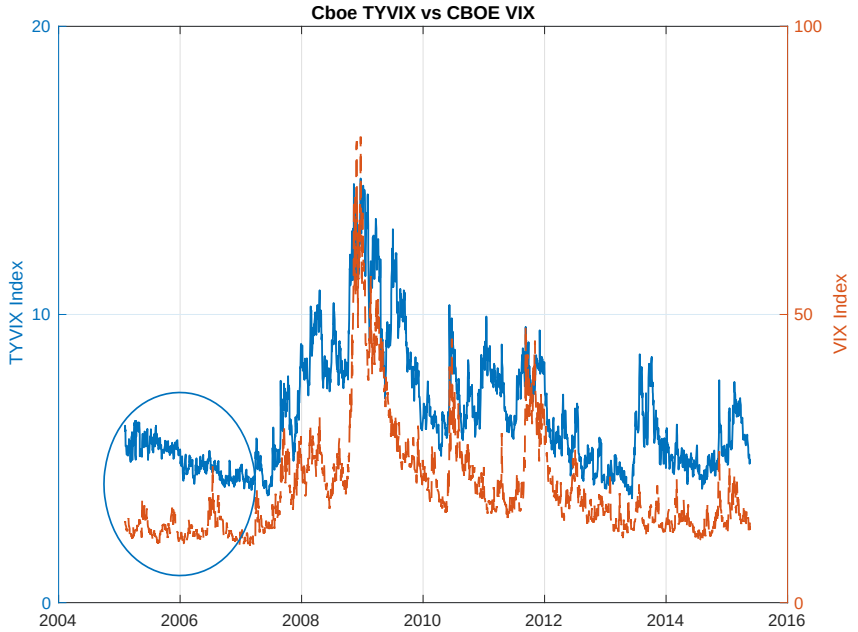
Monetary policy: low interest rates during the first half of the 2000s (also low capital market volatility)



Low capital market volatility—interest rate swaps



Low capital market volatility—equity and treasury (forward looking, "VIX")

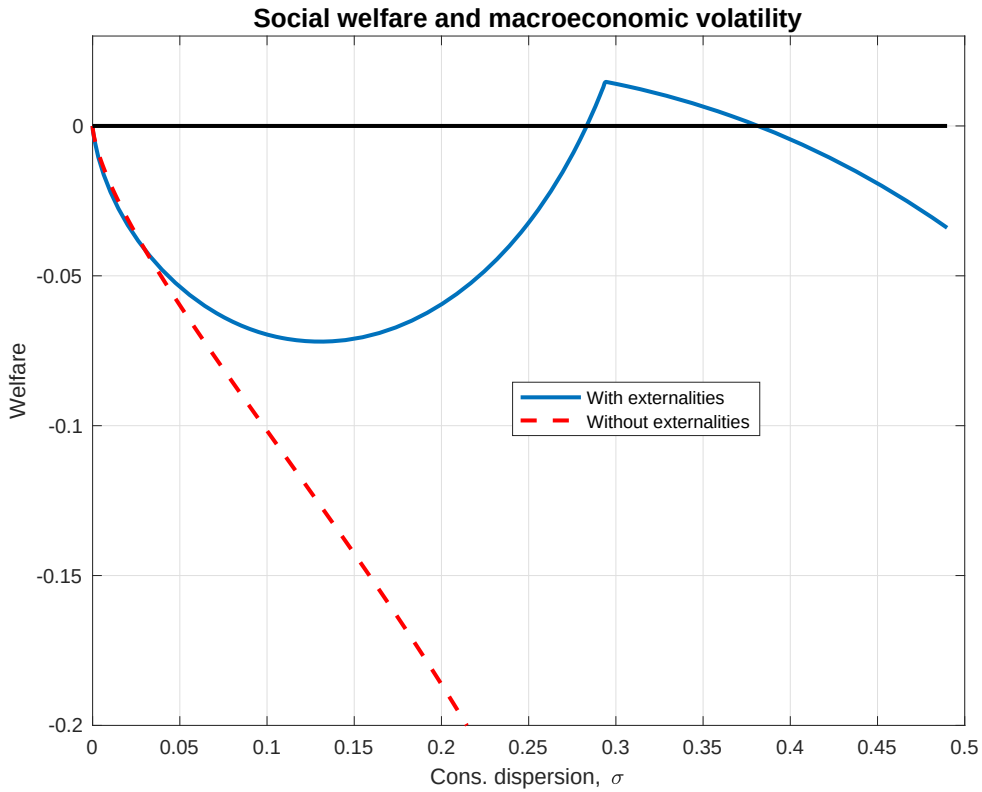


* Cboe TYVIX is currently being replaced by Cboe VIX TLT

Volatility paradox in a simple example

- Macroeconomic volatility and risk-seeking behavior
- Lower volatility leads financiers to take on more risk
  - but risks may materialize and hurt the economy

▷ Society may be better off with non-negative volatility


Social welfare and macroeconomic volatility
Welfare
0
-0.05
-0.1
-0.15
-0.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Cons. dispersion, σ
With externalities
Without externalities

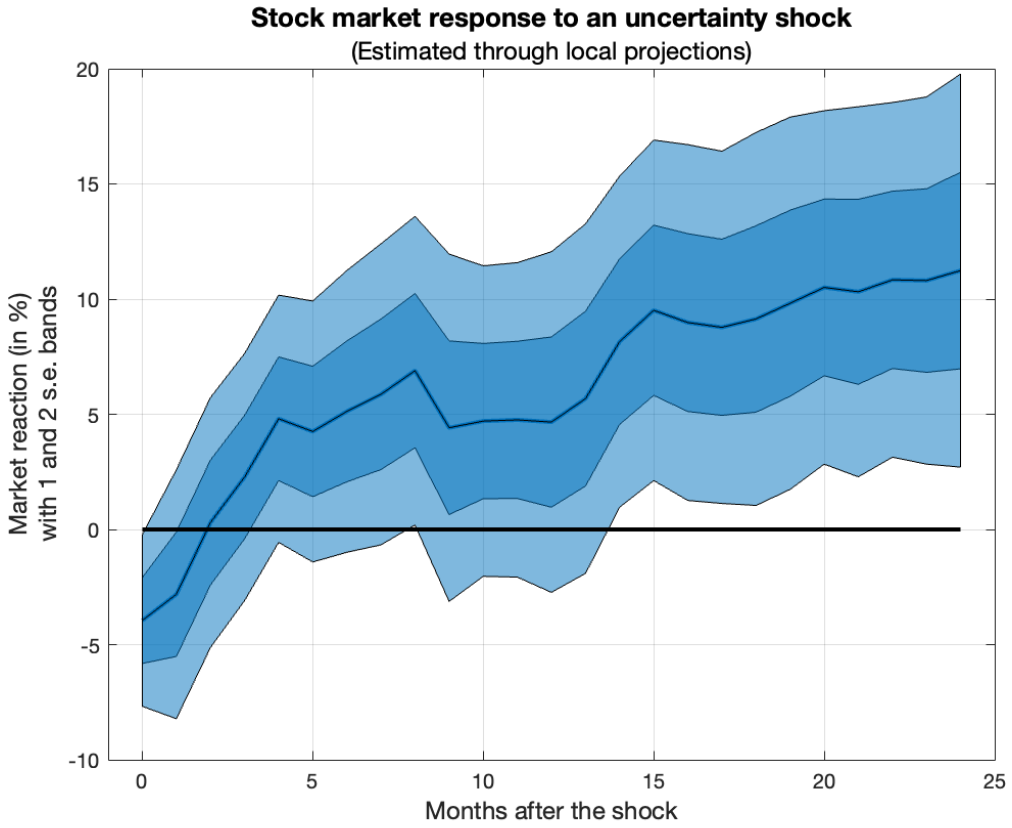
Reacting to uncertainty shocks?

... what should monetary/fiscal policy do following an uncertainty shock?

- Starting point: Seventeen uncertainty shocks identified by macroeconomists (Bloom, 2009)

October 1962	Cuban missile crisis	October 1987	Black Monday
November 1963	Assassination of JFK	September 1990	Gulf War I
August 1966	Vietnam buildup	November 1997	Asian Crisis
May 1970	Cambodia and Kent State	September 1998	Russian, LTCM default
December 1973	OPEC I, Arab-Israel War	September 2001	9/11 terrorist attack
September 1974	Franklin National	July 2002	Worldcom and Enron
November 1978	OPEC II	February 2003	Gulf War II
March 1980	Afghanistan, Iran hostages	August 2007	Credit Crunch
August 1982	Monetary cycle turning point		

- Impulse-response analysis



$\Longrightarrow$ Markets seem to recover quite quickly after the occurrence of an uncertainty shock ...

- Not sure whether this property is due to "endogenous" monetary actions or due to spontaneous market forces

Some critical episodes

Lehman Brothers

- Skipped: please see Mele & Obayashi, 2015

Covid-19

- Skipped: please see my personal website: `https://www.antoniomele.org`
  - `https://www.antoniomele.org/volatility-at-the-time-of-covid-19/`

Inflation + Russia-Ukraine conflict

- Skipped: please see my personal website: `https://it.antoniomele.org`
  - `https://it.antoniomele.org/mercati-in-tempo-di-guerra/`

Thank you