

Springer Finance

Antonio Mele
Yoshiki Obayashi

The Price of Fixed Income Market Volatility

 Springer

Editorial Board

Marco Avellaneda

Giovanni Barone-Adesi

Mark Broadie

Mark Davis

Emanuel Derman

Claudia Klüppelberg

Walter Schachermayer

Springer Finance

Springer Finance is a programme of books addressing students, academics and practitioners working on increasingly technical approaches to the analysis of financial markets. It aims to cover a variety of topics, not only mathematical finance but foreign exchanges, term structure, risk management, portfolio theory, equity derivatives, and financial economics.

For further volumes:
www.springer.com/series/3674

Antonio Mele • Yoshiki Obayashi

The Price of Fixed Income Market Volatility

 Springer

Antonio Mele
Swiss Finance Institute
University of Lugano
Lugano, Switzerland

Yoshiki Obayashi
Applied Academics LLC
New York, NY, USA

ISSN 1616-0533
Springer Finance
ISBN 978-3-319-26522-3
DOI 10.1007/978-3-319-26523-0

ISSN 2195-0687 (electronic)
ISBN 978-3-319-26523-0 (eBook)

Library of Congress Control Number: 2015958378

Mathematics Subject Classification (2010): 91G20, 91G30, 91G40, 91B25

Springer Cham Heidelberg New York Dordrecht London

© Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

The volatility of major asset classes is a key driver of portfolio performance affecting institutional and individual investors alike. Portfolio volatility may be managed by diversification through asset allocation and security selection decisions as well as by derivatives supplying direct exposure to volatility. Of the two main traditional assets—stocks and bonds—volatility derivatives and methodologies underlying their designs and pricing have been well-developed for equity markets, while fixed income markets have lagged in this respect despite its principal role in capital markets. This book fills, or at least aims to significantly narrow, this gap.

While the exposition of this book is of a theoretical nature, its ultimate objective is to serve as a foundation upon which a market for standardized fixed income volatility trading may be built. In fact, some of the interest rate volatility index designs proposed in this book have already been brought to life in the US and in Japan—by far the two largest government bond markets by notional outstanding. The Chicago Board Options Exchange (CBOE), home to the omnipresent CBOE Volatility Index[®] (VIX[®]), launched the CBOE Swap Rate Volatility IndexSM (SRVIXSM) in 2012, and subsequently partnered with the CME Group to launch the CBOE/CBOT 10-Year US Treasury Note Volatility IndexSM (TYVIXSM) in 2013. Across the Pacific, S&P Dow Jones Indices and Japan Exchange Group partnered to launch the S&P/JPX JGB VIX in 2015; a Japanese Government Bond analogue of TYVIX. CBOE listed TYVIX futures in 2014 as its first-ever listed derivative for standardized fixed income volatility trading, and counterpart to its popular VIX for broad equity market volatility trading.

This book presents a unified fixed income volatility evaluation framework and in-depth accounts of its application to four major fixed income asset sub-classes: interest rate swaps, government bonds, time deposits, and credit. It develops model-free, forward-looking volatility indexes for each of these asset types, which involves dealing with disparate market conventions and numerous complexities that are absent when pricing equity volatility. Some of these complexities had long been recognized by practitioners as hurdles for creating VIX-like indexes for interest rate volatility, but left bereft of mathematically rigorous solutions for a number of years,

which presumably contributed to the stunted development of standardized measurement and trading of fixed income volatility until recently.

Our work draws its origins from a series of research notes and implementation details that we developed over the last six years. The present monograph organizes this work in a self-contained fashion by providing the reader with a comprehensive piece with interconnected parts. Our work does not focus on purely mathematical innovations; rather, it relies on existing methods and develops new tools aimed at facilitating contract evaluation in the fixed income space. It is our hope that this work will lead to significant and positive contributions in the world of financial engineering, and will help investors measure and better manage risks arising from fixed income volatility.

This book will appeal to both applied researchers and theorists. Researchers in academia and at financial institutions are the main audience of this book, while advanced students in finance, economics, and mathematics should also find the material useful for further study in the area of asset pricing. Applied researchers will gain access to the mathematically rigorous background required for undertaking empirical research in relatively new topics such as: time series behavior of forward-looking interest rate volatility indexes; interest rate volatility risk-premiums; linkages between volatility and market liquidity; and the impact of macroeconomic developments and monetary policy on fixed income volatility. Theorists will find contributions to an exciting area in asset pricing regarding interest rate volatility evaluation as well as the evaluation of new financial products referenced to forward-looking gauges of interest rate volatility, such as TYVIX futures.

Last but not least, we are very grateful to seven anonymous reviewers for their insightful comments on our work and valuable suggestions, and to Dr. Catriona Byrne at Springer for coordinating the countless moving parts involved in bringing this project to fruition. We also thank Kodjo Apedjinou, Giovanni Barone-Adesi, Ruslan Bibkov, Peter Carr, Praveen Korapaty, Catherine Shalen, and David Wright for comments and suggestions throughout this project, Shihao Yang for excellent research assistance, and audiences at the IAQF (International Association of Quantitative Finance) Thalesian seminar in New York University, Jane Street Capital, Kellogg School of Management, Morgan Stanley Quantitative Risk Modeling Group in New York, the NYU-Stern Volatility Institute, the 7th Annual Risk Management Conference at National University of Singapore, the 4th Annual Global Derivatives in Chicago, the 19th Annual RISK USA Conference in New York, the 1st and the 3rd CBOE Risk Management Conference in Europe (Dublin), the 8th Annual Meeting of the Swiss Finance Institute in Geneva, and a Swiss Finance Institute Knowledge Transfer Workshop in Lugano for remaining comments. However, we retain full responsibility for any remaining omissions or mistakes.

Lugano, Switzerland
New York, USA
September 2, 2015

Antonio Mele
Yoshiki Obayashi

Contents

1	Introduction	1
1.1	Background	1
1.2	From Realized to Expected Fixed Income Volatility	4
1.3	The Right <i>Numéraire</i> and Volatility Pricing	8
1.3.1	Market Risk and Model-Free Pricing	9
1.3.2	Getting the Right Volatility with the Right Model	11
1.4	Scope and Plan of the Book	16
2	Variance Contracts: Fixed Income Security Design	19
2.1	Introduction	19
2.2	Market Numéraires and Volatilities	21
2.3	Interest Rate Variance Swaps	22
2.3.1	Contracts and Model-Free Pricing	22
2.3.2	Log Versus Quadratic Contracts	26
2.3.3	Hedging	28
2.3.4	Constant Gamma Exposure	30
2.4	Implied Volatility Indexes	31
2.4.1	Model-Free Indexes	31
2.4.2	Comparisons to Model-Based Log-Normal and Normal Implied Volatility	32
2.4.3	Index Decompositions	34
2.5	Implementing Basis Point Variance Swaps	35
2.5.1	Incremental Versus Point-to-Point Realized Variance	35
2.5.2	Volatility Risk Premiums	38
2.6	Skew Shifts and the Dynamics of Volatility Indexes	40
2.6.1	Truncations	41
2.6.2	Numerical Experiments and Interpretation of Actual Index Behavior	43
2.7	Jumps	47
Appendix A	Appendix on Security Design and Volatility Indexing	49
A.1	Proof of Proposition 2.2	49

A.2	A Stochastic Multiplier Beyond the Market NumÉraire	51
A.3	Vega and Gamma in Gaussian Markets	51
A.4	Proof of Proposition 2.3	54
A.5	Approximating Indexes	55
A.6	Jumps	57
3	Interest Rate Swaps	59
3.1	Introduction	59
3.2	Risks Regarding Interest Rate Swaps	61
3.2.1	The Annuity Factor	61
3.2.2	Option-Based Volatility Trading	63
3.3	Interest Rate Swap Variance Contracts	66
3.3.1	Risks and Spanning Derivatives	67
3.3.2	Contract Designs	68
3.3.3	Pricing	70
3.3.4	Marking to Market	71
3.3.5	Hedging	72
3.3.6	Links to Constant Maturity Swaps	76
3.3.7	Physical Swap Settlement and Variance Contracts	76
3.3.8	Multiple Curves	77
3.4	Trading Strategies	78
3.4.1	Spot Trading Through IRV Swaps	78
3.4.2	Spot Trading Through Standardized IRV Swaps	81
3.4.3	Forward Trading	82
3.5	Interest Rate Swap Volatility Indexes	83
3.5.1	Basis Point Volatility Index	83
3.5.2	Percentage Volatility Index	84
3.5.3	Experiments	84
3.5.4	Jumps	87
3.6	Swap Versus Equity Variance Contracts and Indexes	87
3.7	Index Implementation	89
3.7.1	A Numerical Example	90
3.7.2	Historical Performance	93
Appendix B	Appendix on Interest Rate Swap Markets	97
B.1	P&L of Option-Based Volatility Trading	97
B.2	Spanning IRS Variance Contracts	103
B.3	Hedging	107
B.4	Constant Maturity Swaps	111
B.5	The Contract and Index in the Vasicek Market	113
4	Government Bonds and Time-Deposits	125
4.1	Introduction	125
4.2	Government Bonds	129
4.2.1	Pricing Spot Volatility	129
4.2.2	Basis Assets	134
4.2.3	Percentage <i>Price</i> Volatility	136

- 4.2.4 Basis Point *Price* Volatility 138
- 4.2.5 Marking to Market 139
- 4.2.6 Replication 139
- 4.2.7 Forward Price Adjustments 141
- 4.2.8 Model-Free Measures of Basis Point *Yield* Volatility 142
- 4.2.9 Certainty Equivalent Bond Prices as Expectations
of Forward Prices 145
- 4.2.10 Early Exercise and Futures Corrections 148
- 4.2.11 Implementation Example 153
- 4.2.12 Jumps 157
- 4.3 Time Deposits 157
 - 4.3.1 The Underlying Risks 157
 - 4.3.2 Variance Contracts and Volatility Indexes 158
 - 4.3.3 Yield Volatility 160
 - 4.3.4 American Future Corrections 161
 - 4.3.5 Implementation Example 165
 - 4.3.6 LIBOR Variance Contracts and Volatility Indexes 168
- 4.4 Maturity Mismatch 170
 - 4.4.1 Government Bonds 170
 - 4.4.2 Time Deposits 176
 - 4.4.3 Alternative Characterizations of Variance Contracts
and Indexes 182
 - 4.4.4 Tilting the Variance Payoff 184
- 4.5 Index Design with Heterogeneous Market Data 185
 - 4.5.1 Sandwich Combinations 186
 - 4.5.2 Rolling Indexes 188
- Appendix C Appendix on Government Bonds and Time Deposit
Markets 189
 - C.1 The Equity VIX with Stochastic Interest Rates 189
 - C.2 Naïve Model-Free Methodology and Bias in Vasicek’s
Market 191
 - C.3 Marking to Market 193
 - C.4 Replication of Variance Swaps 193
 - C.5 Estimates Based on Forward Price Approximations 195
 - C.6 Certainty Equivalence, and Existence of Basis Point
Yield Volatility 197
 - C.7 Illustrations with a Stochastic Volatility Model 199
 - C.8 The Future Price in Vasicek’s Model 202
 - C.9 Future and Forward LIBOR Options in Vasicek’s Model 202
 - C.10 The Impact of Early Exercise Premiums and Maturity
Mismatch 206
- 5 Credit 211**
 - 5.1 Introduction 211
 - 5.2 Existing Credit Trading Practices 213

5.2.1	Assumptions	214
5.2.2	CDS Indexes	214
5.2.3	CDS Index Options	215
5.3	Credit Variance Contracts	218
5.3.1	Percentage	218
5.3.2	Basis Point	220
5.3.3	Marking to Market	221
5.4	Credit Volatility Indexes	225
5.4.1	Definitions	225
5.4.2	Forward Premium Adjustments	226
5.4.3	Differences with Respect to Other Fixed Income Volatility Gauges	226
5.4.4	Implementation Example	227
5.4.5	Index Design Through Option Cycles	230
5.5	Post “Big-Bang” Conventions and Index Adjustments	230
5.5.1	Index Values Under Constant Hazard Rates	231
5.5.2	Forward Positions	232
5.5.3	Option Payoffs and Evaluation	233
5.5.4	Index Corrections	236
Appendix D	Appendix on Credit Markets	237
D.1	Preliminary Facts Concerning CDS Indexes	237
D.2	Spanning Credit Variance Contracts	238
D.3	Hedging	242
References	247

Information About the Authors

Antonio Mele holds a Senior Chair at the Swiss Finance Institute and is a Professor of Finance at the University of Lugano, after a decade spent as a tenured faculty at the London School of Economics. He is also a Research Fellow for the Financial Economics program at the Centre for Economic Policy Research (CEPR) in London. He holds a PhD in Economics from the University of Paris. His work spans a variety of fields in financial economics, pertaining to capital market volatility, interest rates and credit markets, macro-finance, capital markets and business cycles, and information in securities markets, and has appeared in journals such as the *Journal of Financial Economics*, the *Review of Economic Studies*, the *Review of Financial Studies*, and the *Journal of Monetary Economics*.

His work outside academia includes developing fixed income volatility indexes for Chicago Board Options Exchange and S&P Dow Jones Indices. He is currently a member of the *Securities and Markets Stakeholder Group* of the European Securities Markets Authority (ESMA), the supra-national supervisor of European financial markets. At ESMA, he is also a member of the Group of Economic Advisers.

Yoshiki Obayashi is a managing director at Applied Academics LLC in New York. The company specializes in developing and commercializing ideas emanating from a growing think tank of academic researchers selected on the basis of their work's relevance to practice in the finance industry. His most recent projects range from running systematic trading strategies for funds to developing fixed income volatility indexes for Chicago Board Options Exchange and S&P Dow Jones Indices.

Yoshiki Obayashi previously managed US and Asian credit portfolios for a proprietary fixed-income trading group at an investment bank. He holds a PhD in Finance and Economics from Columbia Business School.

Chapter 1

Introduction

1.1 Background

On the 6th and 7th of August 2008, the Bank of England's Monetary Policy Committee convened to decide on the course of its monetary policy. The economic environment during the few weeks preceding Lehman Brothers' collapse made this a difficult task. The inflationary pressures mounting in the UK pointed towards higher interest rates, yet the fragility of the global financial system caused by the subprime mortgage crisis, and the UK's substantial exposure to it, pointed towards a policy of low interest rates. The Committee considered three polarized scenarios: an immediate increase in the Bank rate, a cut, and a continuation of the status quo. The majority of the Committee voted in favor of keeping the Bank rate unchanged, with one member voting for an increase and another for a decrease (Bank of England 2008). Many other outcomes were arguably conceivable during the summer of 2008.

How can one hedge against, or express views on, uncertainties surrounding interest rate movements? Interest rate volatility has traditionally been traded through option-based strategies such as a straddle, which is comprised of long positions in both a payer and a receiver swaption, which may pay off if swap rates experience heightened volatility during the life of the trade. However, a common frustration is that such strategies do not necessarily lead to profits consistent with directional volatility views. As with equity option straddles, swaption straddles suffer from "price dependency," which is the sensitivity of returns to not only the absolute movement of the underlying, i.e. volatility, but also to its direction. Together with other market forces, this problem likely contributed to the emergence of variance swap contracts in equity markets that better align directional views with payoffs, as well as dedicated volatility derivatives such as those tied to the Chicago Board Options Exchange's ("CBOE") Volatility Index ("VIX").

Variance swap contracts are defined as forward agreements whereby one party receives payments tied to the realized variance of the price of a security over a given horizon. They overcome challenges that arise when hedging volatility using options (a straddle, say) by insulating the volatility component of a "dirty" P&L that is muddled by path dependency. The CBOE VIX is linked to the fair value of a variance