A Theory of Debt Accumulation and Deficit Cycles

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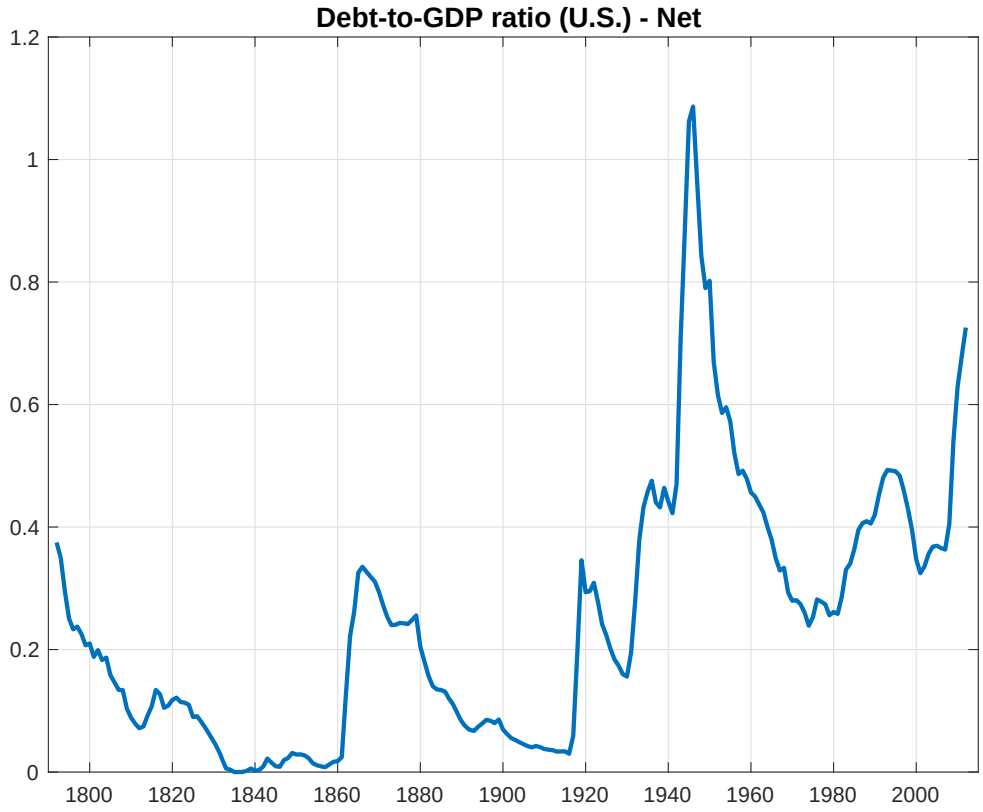
"Our 2010 paper found that, over the long term, growth is about 1 percentage point lower when debt is 90 percent or more of gross domestic product."

Carmen Reinhart and Kenneth Rogoff, April 26, 2013, The New York Times

Why do nations accumulate debt?

- Enormous literature
- Tax smoothing arithmetics explains they shouldn't (Barro, 1979)

But they do


Debt-to-GDP ratio (U.S.) - Net
1.2
1
0.8
0.6
0.4
0.2
0
1800
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000

Fiscal tipping points?

Some explanations

- Governments may reflect electoral bases that value either high expensens or low taxes, or both.
  - Buchanan and Wagner (1977): fiscal illusion doctrine—electors have limited knowledge about the intertemporal implications of their preferences for deficits
- Other political economy theories: governments face fully rational voters (Alesina and his co-authors)
  - To illustrate, a policymaker in office may not fully internalize the debt burden while facing a probability of not being re-elected, or in the presence of political polarization
- Deficit bias

This paper

- "Austerity" (Giavazzi and Pagano, 1990)
- A missing piece: When do governments implement austerity?
- This paper develops a model where governments display preferences for deficits but are also rationally concerned about costly default
  - Costs of implementing austerity are determined endogenously

Predictions

1. When debt is small compared to the size of the economy, governments accumulate debt for some period

2. Fiscal austerity may arrive too late—at about 80-90% of debt levels that trigger default events

3. Lenght of debt accumulation regime increases with

   (a) growth rate of the economy
   (b) government short-sightedness
   (c) macroeconomic stability
   (d) expected time defaulted governments may re-gain access to capital markets
   (e) expected severity of austerity measures
   (f) debt market liquidity

Predictions cont'd

4. Probability of default increases with

   (a) governments short-sightedness
   (b) ease at which defaulted governments re-gain access to capital markets (leading to serial defaulting)

5. Liquidity conditions affect default probabilities in an ambiguous way:

   (a) liquidity support to distressed debt lowers spreads but,
   (b) leads to future higher debt and spreads

6. New puzzle: Probability of default decreases with macroeconomic uncertainty

   - Higher macro volatility leads governments to anticipate austerity
   - This effect is strong and dominates mechanical effects (high vol $\longrightarrow$ high default prob)
   - Public debt version of the "volatility paradox"

Additional links to the literature

- Analysis of fiscal tipping points: related to liquidity management and dividend policy + agency problems in the corporate finance literature
  - Jeanblanc-Picqué and Shirayev (1995) & Radner and Shepp (1996) (extended to agency problems by DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin and Rochet, 2007)

  To draw an analogy, whereas, in this literature, problem is one of a private firm managing cash distribution under liquidity constraints (or one of a potential conflict of interest between a principal and an agent), our problem is one of a government managing primary deficits under liquidity constraints
  - In both cases, some agent takes some action (dividend or payment, in the current literature; fiscal surpluses, in this paper) as soon as an underlying endogenous variable hits a threshold

  - Due to the nature of our problem, our analysis relies on discrete interventions—that is, ours is not a singular stochastic control problem

- A variant of our model also deals with strategic default

  - default boundaries dealt then with as in the standard literature started by Leland (1994)—with the added complication related to the fiscal tipping points

Outline

1. Preferences for deficits and the cost of national debt
2. Fiscal tipping points
3. Spreads

1. Preferences for deficits and the cost of national debt

Output and debt

- Output growth is I.I.D.:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t,$$

  where W_t is a standard BM

- Government issues short-term debt to finance deficits. Constraint is $\dot{D}_t = -S_t + iD_t$, where i is the short-term rate and S_t is government surplus
  - Debt-to-GDP ratio, δ_t, is solution to

$$d\delta_t = -\left(s_t + \kappa\right)\delta_t dt - \sigma\delta_t dW_t, \tag{1}$$

  where $\kappa \equiv \mu - i - \sigma^2$, and s_t is the surplus-to-debt ratio, $s_t = \frac{S_t}{D_t}$

Governments' preferences and policy

- Preferences for primary deficits—may result from
  - attempt to respond to electoral basis—to illustrate, voters may neglect the effects that debt burden may impose in the future
  - voters would value governments that assign high weight to deficits
    - up to some (endogenous) probability of default (determined in the paper)

- Government seeks to maximize deficits for any given level of debt

$$V(\delta_t) = \inf_{s_u \in [s^1, s^2]} E_t \left[\int_t^{\infty} e^{-\rho(u-t)} s_u du \right], \tag{2}$$

under the debt accumulation constraint in Eq. (1)

Comments

- Eq. (2) doesn't describe a singular stochastic control problem. Due to the nature of the problem, our model relies on **discrete interventions**
  - To illustrate, we might have considered a model in which a government accumulates debt until it optimally decides for austerity whereby, and unlike in this paper, a fiscal surplus would then occur at an infinite rate
  - In this case, debt would be driven by a regulated Brownian motion
  - But a fiscal surplus run at an infinite rate in a time of austerity does not seem to be plausible

Comments cont'd

- In most of this presentation, i is exogenously fixed, but Section 5 of the paper contains the model predictions resulting when the cost of borrowing is state-dependent

- Deficits can be high either due to large government expenses or low taxation

- s^1 and s^2: bounds on the government feasible actions, e.g., international compacts

Comments cont'd

- Time preference ρ determines how myopic the government is. Might result from political competitiveness
  - Example: at any instant of time, government faces a joint probability of being confronted in a snap election and losing that election equal to a constant p
  - Value function satisfies $\mathcal{L}V + s - kV + p(\underline{V} - V) = 0$, where k is the discount rate and $\underline{V}$ is the value from losing the snap election
  - Under reg conditions, this is Eq. (2) with $\rho = k + p$

Policy

- Bellman equation

$$
\begin{aligned}
0 &= \inf_{s} \left[\mathcal{L}V\left(\delta\right) - \rho V\left(\delta\right) + s \right] \\
&= \frac{1}{2}\sigma^2\delta^2 V''\left(\delta\right) - \kappa\delta V'\left(\delta\right) - \rho V\left(\delta\right) + \inf_{s} \left[s\left(1 - V'\left(\delta\right)\delta\right) \right],
\end{aligned}
$$

  subject to a number of boundary conditions

- Technically it's a Stefan problem
  - Need find value for δ that triggers a switch in the equation satisfied by $V\left(\delta\right)$
  - Search for $\hat{\delta} : V'(\hat{\delta})\hat{\delta} = 1$ (free boundary)

- Economically this value $\hat{\delta}$ is the **fiscal tipping point**—austerity trigger

Proposition I. (Fiscal tipping point and government utility costs). *There exists a threshold value of the debt-to-GDP ratio* $\hat{\delta}$ *such that the government runs a deficit* $s_t = s^1$ *for all* $\delta_t < \hat{\delta}$, *and a surplus* $s_t = s^2$ *for all* $\delta_t > \hat{\delta}$. *The utility costs satisfy* $V(\delta_t) = V_{\mathcal{D}}(\delta_t)\,\mathbf{1}_{\delta_t<\hat{\delta}} + V_{\mathcal{S}}(\delta_t)\,\mathbf{1}_{\delta_t>\hat{\delta}}$, *where*

$$V_{\mathcal{D}}(\delta) = \frac{s^1}{\rho} + A_{\mathcal{D}2}\delta^{m_{\mathcal{D}2}}, \quad V_{\mathcal{S}}(\delta) = \frac{s^2}{\rho} + A_{\mathcal{S}1}\delta^{m_{\mathcal{S}1}} + A_{\mathcal{S}2}\delta^{m_{\mathcal{S}2}},$$

for some constants $\hat{\delta}$, $m_{\mathcal{D}2} > 0$, $m_{\mathcal{S}1} < 0$, $m_{\mathcal{S}2} > 0$, $A_{\mathcal{D}2} < 0$, $A_{\mathcal{S}1} < 0$, $A_{\mathcal{S}2} > 0$.

- Result is general
- How to get to default? What happens after default?

Default and re-entry

- **Default**
  - Assume exogenously given liquidity crisis
  - Also solve a model combining liquidity crisis + strategic default (in this case Prop I changes a bit: see below)
- **After default**
  - Will re-entry with some exogenously given probability
- **Notation**
  - Government costs at default: $\bar{V} = V(\bar{\delta})$, where $\bar{\delta}$ is the default boundary

Marginal cost of debt

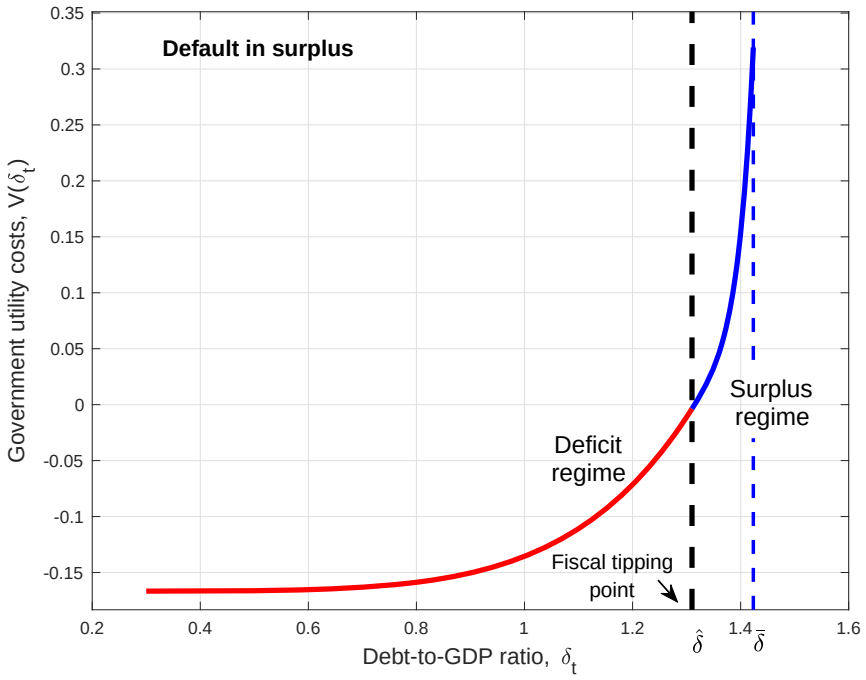
- The value function, $V(\delta_t)$, is the minimized expected value of future surpluses: government utility costs when debt-to-GDP ratio is δ_t

- Utility costs are increasing in δ_t: the closer to default, the higher the costs; otherwise, should be much smaller and relatively insensitive to changes in δ_t

- Suppose a government is running deficit. At each point in time,
  - may either raise debt by some $\delta\Delta$: utility costs would increase by $V(\delta + \delta\Delta) - V(\delta)$
  - or immediately run a surplus of the same amount: would cost just $\delta\Delta$
  - Therefore, policy is to keep on raising debt as long as

$$V(\delta + \delta\Delta) - V(\delta) < \delta\Delta$$

- Taking the limits of the previous condition for $\Delta \longrightarrow 0$ leaves $\mathcal{V}(\delta) \equiv V'(\delta)\,\delta < 1$
  - Governments run deficits when $\mathcal{V}(\delta)$, the mg cost of raising debt, is less than the mg cost of a primary surplus
  - Fiscal tipping points = values of δ s.t. mg costs of raising debt = costs of entering into austerity



- Governments enter into austerity when the marginal costs of running debt become too large—these costs are endogenous in the model
- Run deficits for all $\delta : V'(\delta)\,\delta < 1$ and surpluses for all $\delta : V'(\delta)\,\delta > 1$

Some evidence

α	A_α	B_α	R^2	Tip
95%	0.088	0.842	0.27	3%
	(7.21)	(9.06)		
90%	0.178	0.743	0.06	14%
	(7.149)	(3.72)		
85%	0.168	0.679	0.05	14%
	(6.922)	(3.41)		
80%	0.220	0.671	0.03	20%
	(8.01)	(2.86)		
75%	0.211	0.605	0.01	20%
	(7.82)	(2.59)		
70%	0.280	0.420	0.01	28%
	(9.30)	(1.60)		

TABLE 1. Estimation of an asymmetric Fiscal Reaction Function for the U.S. The table reports estimates of the coefficients A_α and B_α (with t-stats in parenthesis) and R^2 in the following regression: $Tip_t(\alpha) = A_\alpha + B_\alpha\left(\delta_t - Q_{\delta,t}(\alpha)\right) + e_t(\alpha)$, where $Tip_t(\alpha) \equiv \mathbb{I}_{S_t > Q_{S,t}(\alpha)}$, $\mathbb{I}_{S_t > Q_{S,t}(\alpha)}$ is an indicator that takes a value equal to one when S_t, the primary surplus over GDP, is larger than $Q_{S,t}(\alpha)$, δ_t is the debt-to-GDP ratio, $e_t(\alpha)$ is an error term and, finally, $Q_{X,t}(\alpha)$ denote the α-quintile of a variable X at time-t, estimated through the previous ten years of data. Data are yearly and cover the sample from 1792 to 2012, for a total of 221 observations. The column labeled "Tip" reports the fraction of time $Tip_t(\alpha) = 1$ in this sample.

Some details

Default

- Governments may finance total deficit through new debt issuance, provided additional debt is less than a fixed proportion ℓ of GDP. Assume investors are stuck into debt for an arbitrary $\bar{\varepsilon} > 0$. Default boundary is

$$\bar{\delta} = \frac{\ell e^{(s^1+\kappa-\bar{\iota})\bar{\varepsilon}}}{i+\bar{\iota}-s^1} \tag{3}$$

- Also consider alternative measures of debt limits: the Natural Public Debt Limit

$$\bar{\delta}_{\mathrm{n}} = \frac{\bar{s}}{\bar{\iota}_{\mathrm{n}} - \bar{\mu}}$$

  Conclusions are similar but not the same

- Other mechanisms: Collard, Habib and Rochet (2015)

- Leave the task of calibrating the model to alternative definitions of debt limits to future empirical investigations

Re-entry

- After default, government is stuck into a no-deficit policy
- May re-entry with constant intensity $= \vartheta$
- Upon re-entry, a fraction $1 - \gamma$ is forgiven

Default costs

- A proportional one, $\xi\overline{\delta}$ at default for some constant ξ—burden while dealing with bankruptcy: litigation costs, international stigma, loss in popularity related to trade embargoes
  - For example, Panizza and Borensztein (2009) explain that the political consequences of default are quite significant for incumbent governments, and that these costs might be even more severe than the direct economic costs related to financial exclusion (introduced next), which are likely to be short-lived (in our model, $\vartheta > 0$)

- Costs related to financial exclusion $= \epsilon$ per unit of time—burden inherent in building up the new image needed to re-entry the markets
  - For example, ϵ may be a small surplus-over-debt at default stored and distributed to creditors

- Consider a discrete interval of time, Δt. At each point in time during the exclusion period, $t \geq \tau$, the government utility costs are equal to the imminent costs, $\epsilon \Delta t$, plus the same costs expected for the next period, viz

$$\mathcal{C}_d(\bar{\delta}) = \epsilon \Delta t + e^{-\rho \Delta t}[(1 - e^{-\vartheta \Delta t})V(\gamma \bar{\delta}) + e^{-\vartheta \Delta t}\mathcal{C}_d(\bar{\delta})].$$

- We have (boundary condition)

$$\bar{V} = \lim_{\delta \to \bar{\delta}} \mathcal{C}(\delta), \quad \mathcal{C}(\delta) \equiv \underbrace{\frac{\epsilon}{\rho + \vartheta} + \frac{\vartheta}{\rho + \vartheta} V(\gamma \delta)}_{\equiv \mathcal{C}_d(\delta)} + \xi \delta \tag{4}$$

  where the first term on the R.H.S. is the utility cost the government incurs during the exclusion period

Strategic default

Government plans

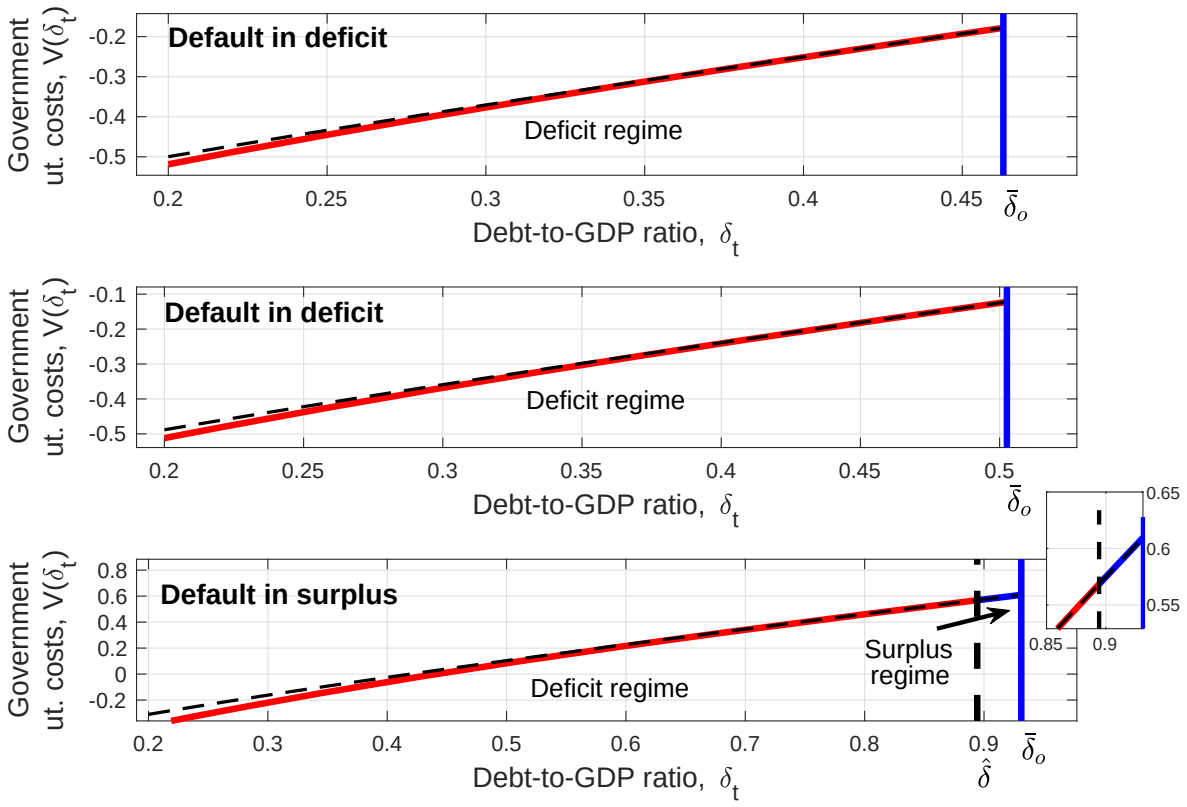
- Government now minimizes future expected surplus-to-debt ratios while also timing default,

$$V\left(\delta_t\right) = \inf_{\tau} \inf_{s_u \in [s^1, s^2]} E_t \left[\int_t^{\tau} e^{-\rho(u-t)} s_u du + e^{-\rho(\tau - t)} \mathcal{C}\left(\delta_\tau\right) \right],$$

  where $\mathcal{C}\left(\delta_\tau\right)$ are the costs of defaulting, taken to be the same as in the exogenous default model (see Eq. (4))

Proposition II. (Default boundaries). *Government utility costs are given by the same function $V(\delta_t)$ in Proposition I, but subject to $\neq$ boundary conditions. Under conditions, the government policy is to default at some finite $\overline{\delta}_o$, provided $\overline{\delta}_o < \overline{\delta}$, where $\overline{\delta}$ is the exogenous default boundary in (3), or to still remain in office until δ_t reaches $\overline{\delta}$.*

Utility costs - strategic default

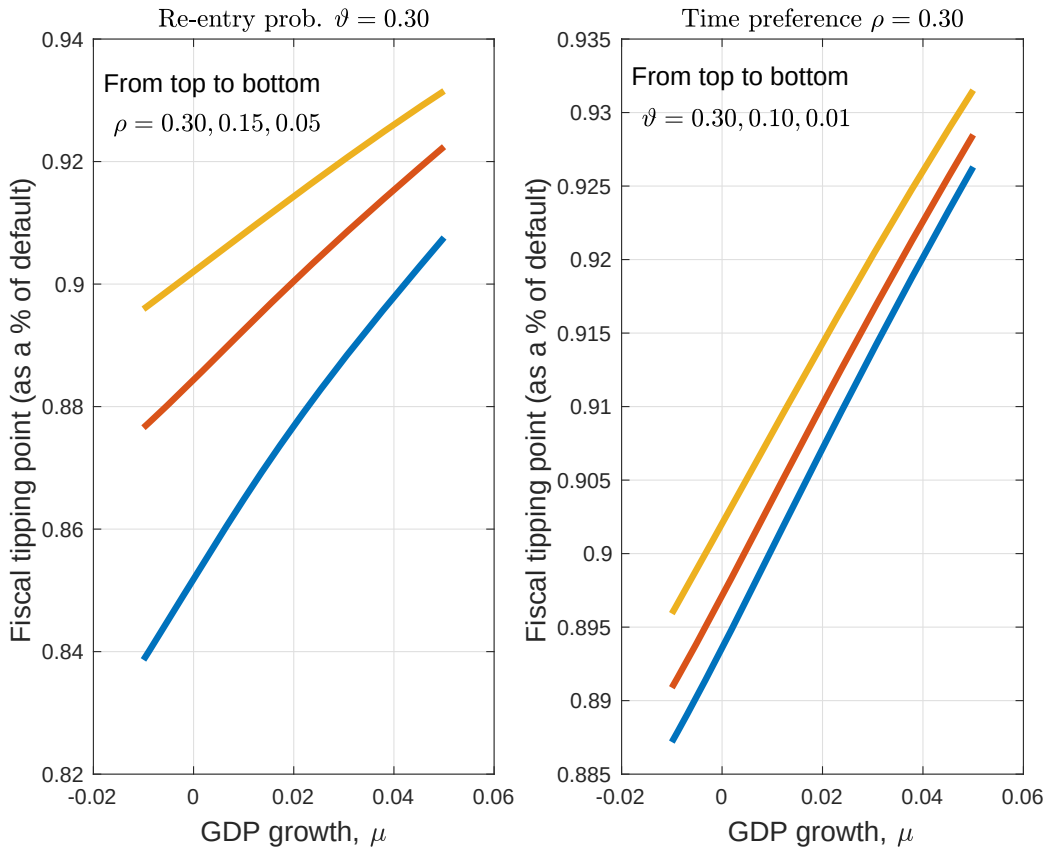


Some predictions

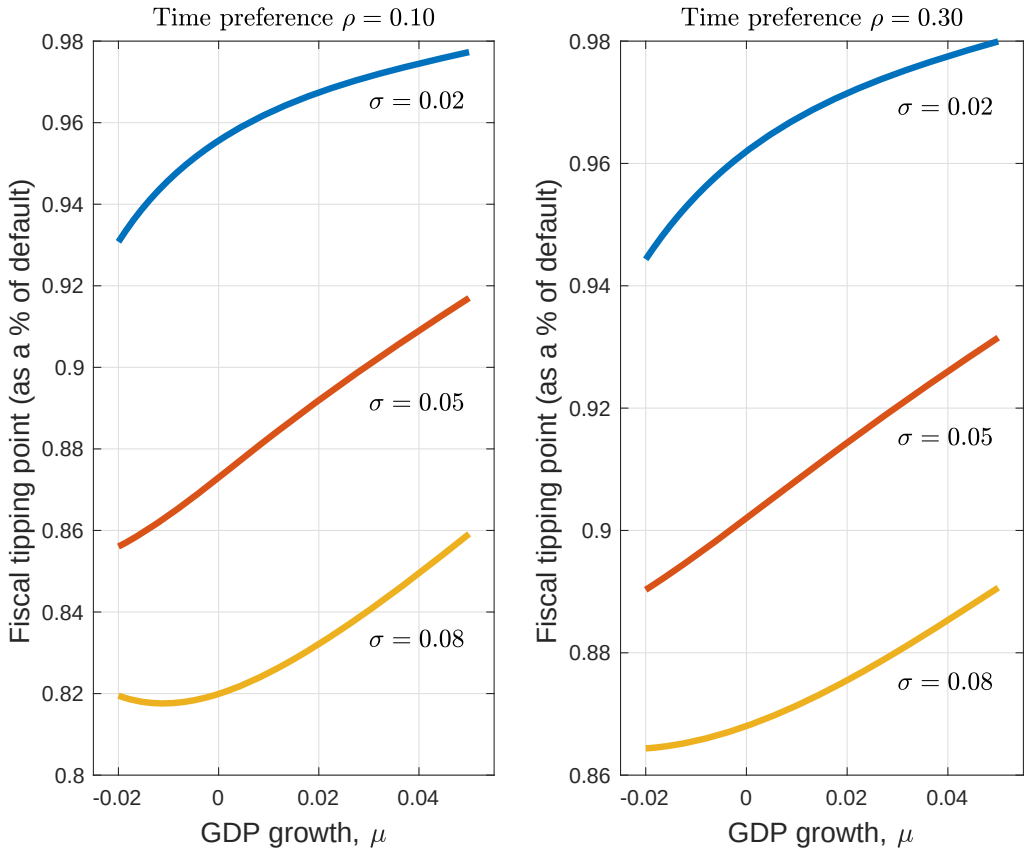
- Default occurs in deficit regimes when μ is low enough
- Default boundary increases with austerity costs
- Default boundary lowers with prob of re-entry: channel to "serial defaulting"
- The higher ρ, the higher the default boundary
  - Gvts now value the chance to re-entry financial markets after default

2. Fiscal tipping points

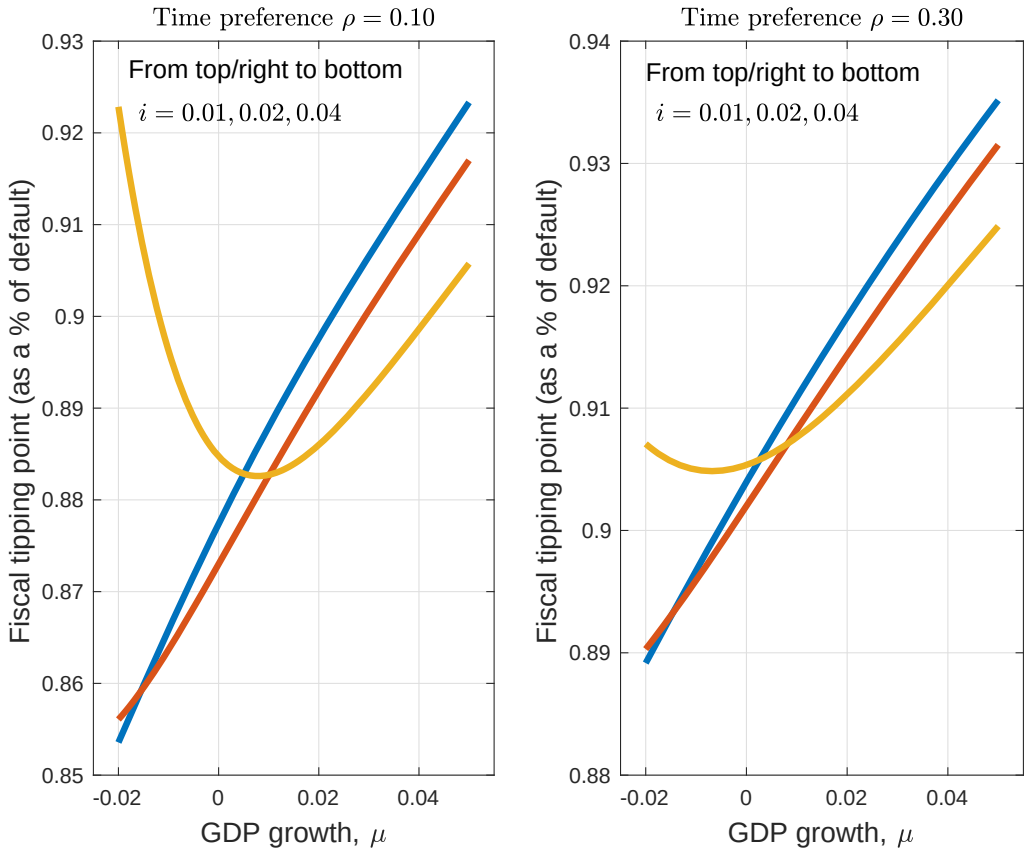
Short-sightedness and re-entry probs



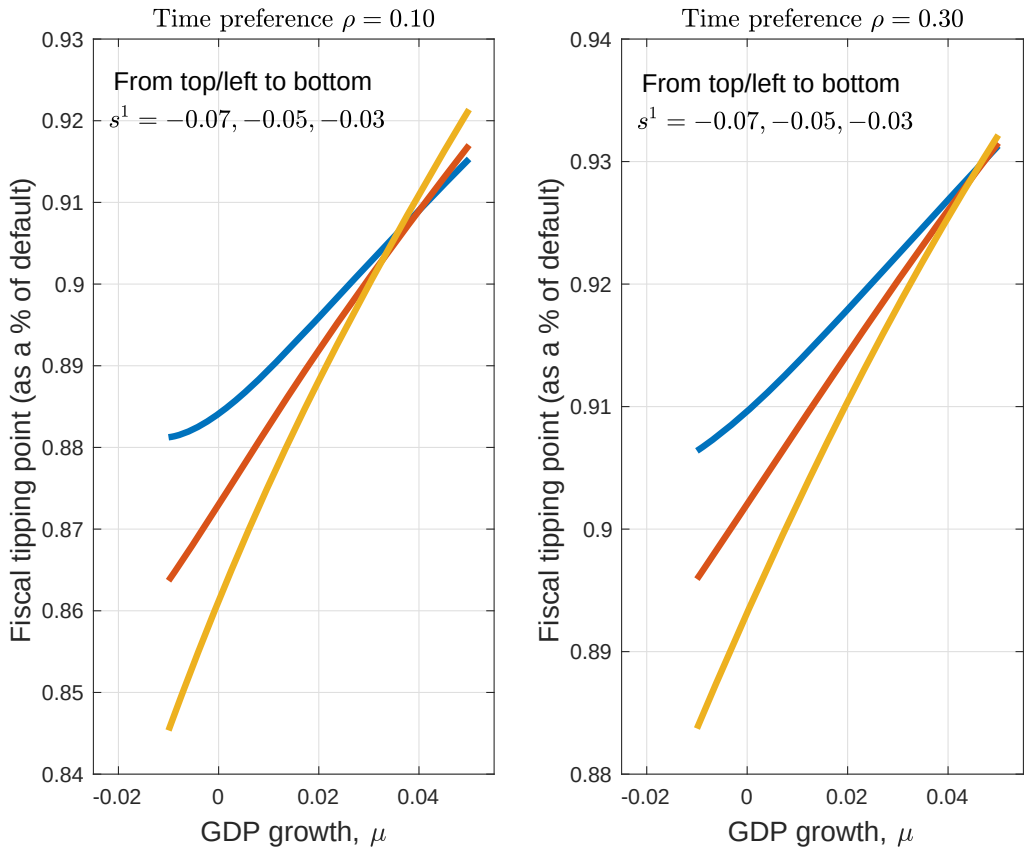
Macroeconomic volatility



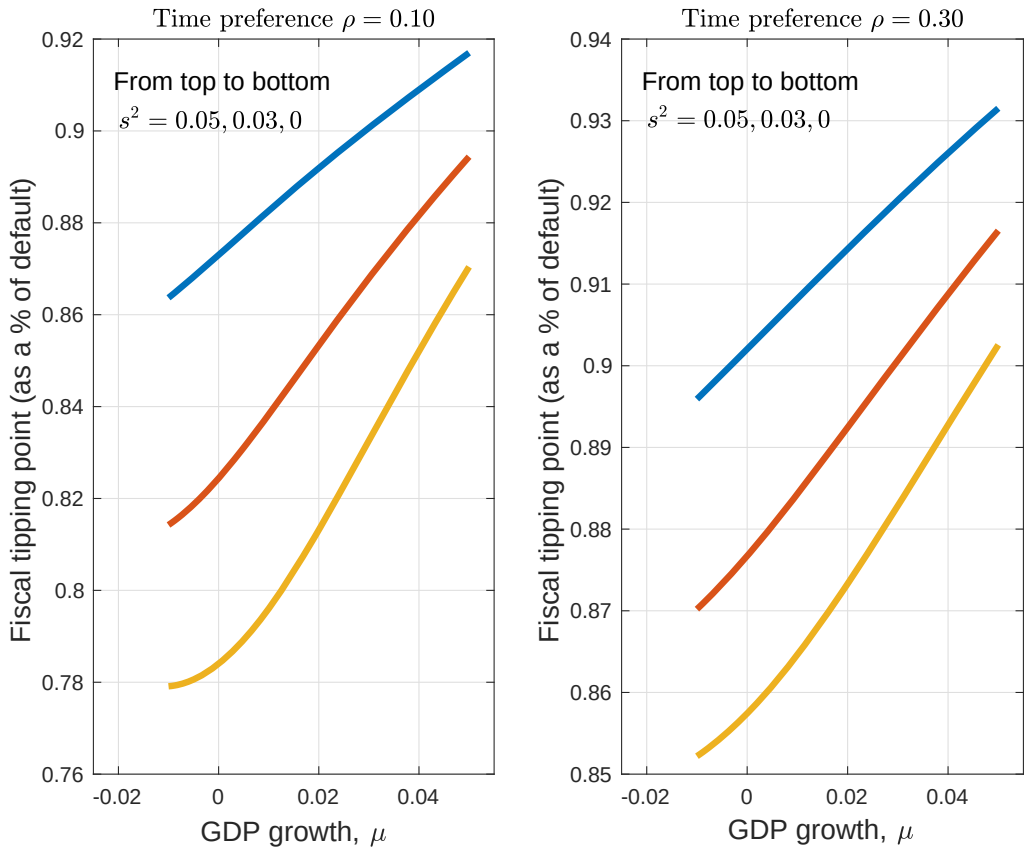
Cost of debt - state independent in this slide



Budget size: deficit



Budget size: surplus



Distortionary policies

- Assess how a given distortionary policy would affect the dynamics of debt and fiscal tipping points

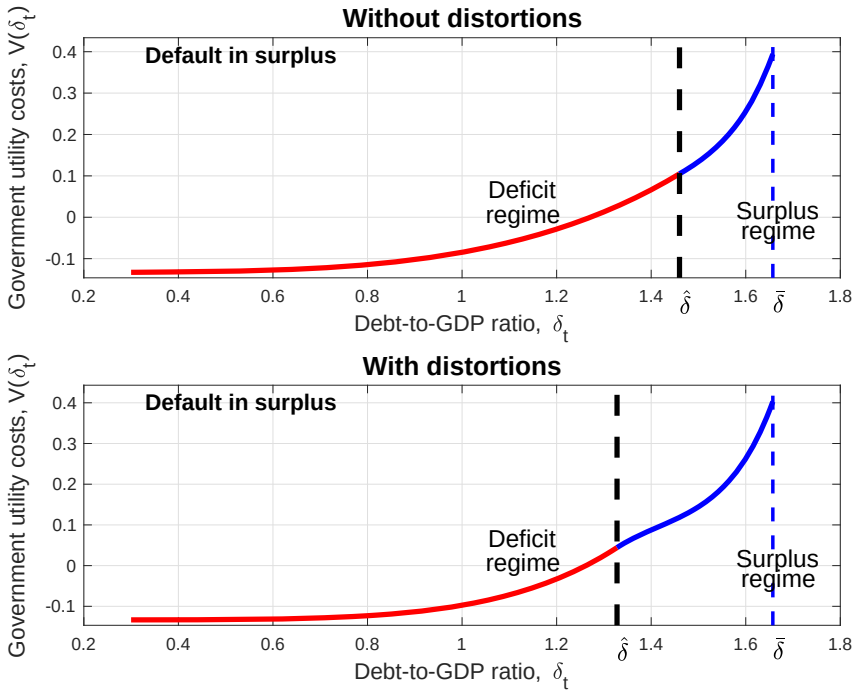
- Two examples of distortionary policies considered in the paper
  - Growth depends on whether the government is in the deficit or in the surplus regime
  - Interest rates are state-dependent

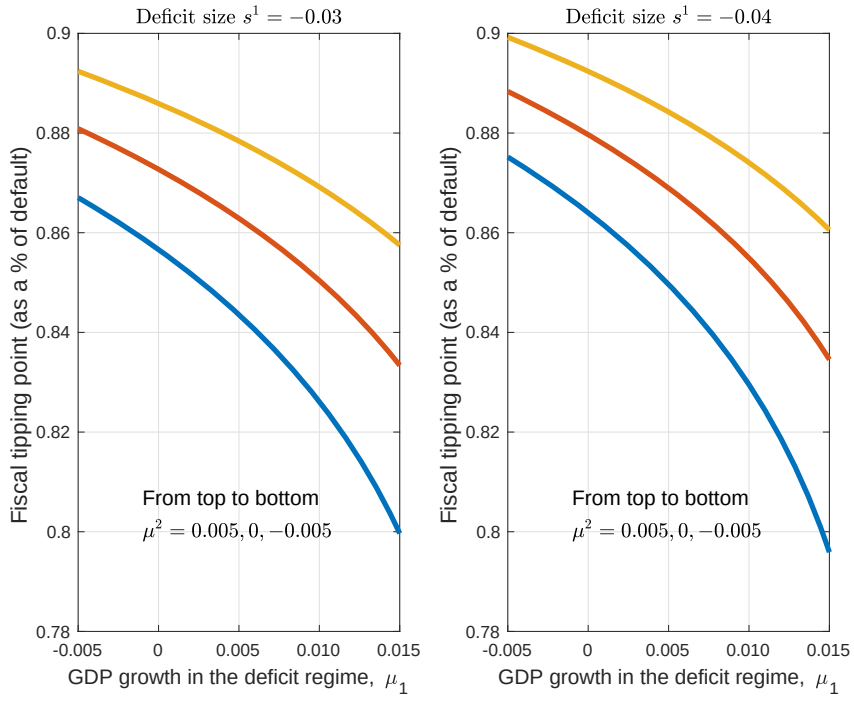
- Bellman equation now is

$$0 = \frac{1}{2}\sigma^2\delta^2 V''(\delta) - (s(\delta) + \kappa(\delta))\,\delta V'(\delta) - \rho V(\delta) + s(\delta)\,. \tag{5}$$

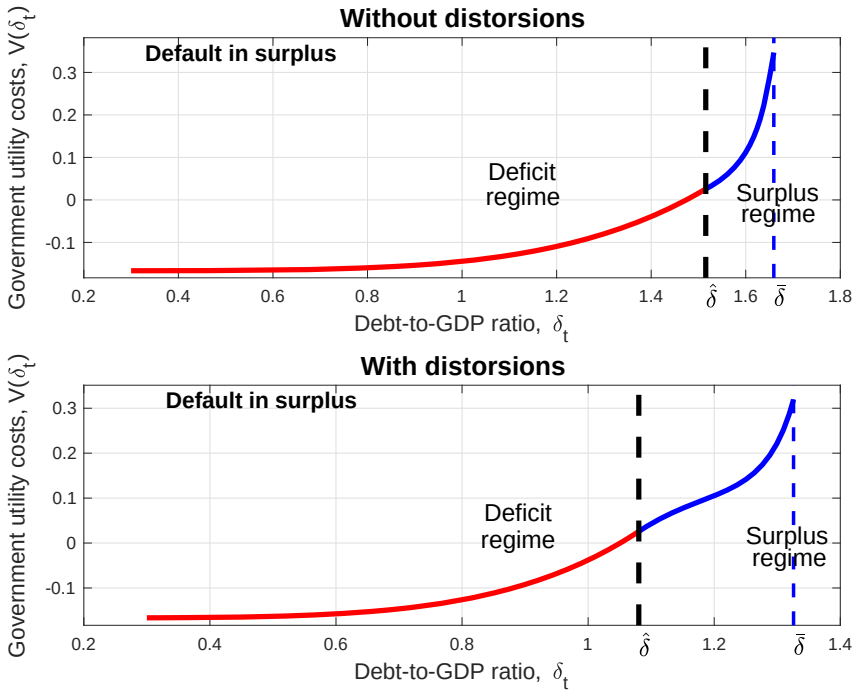
  It is as if government chooses growth and/or the cost of capital whilst choosing its tipping points

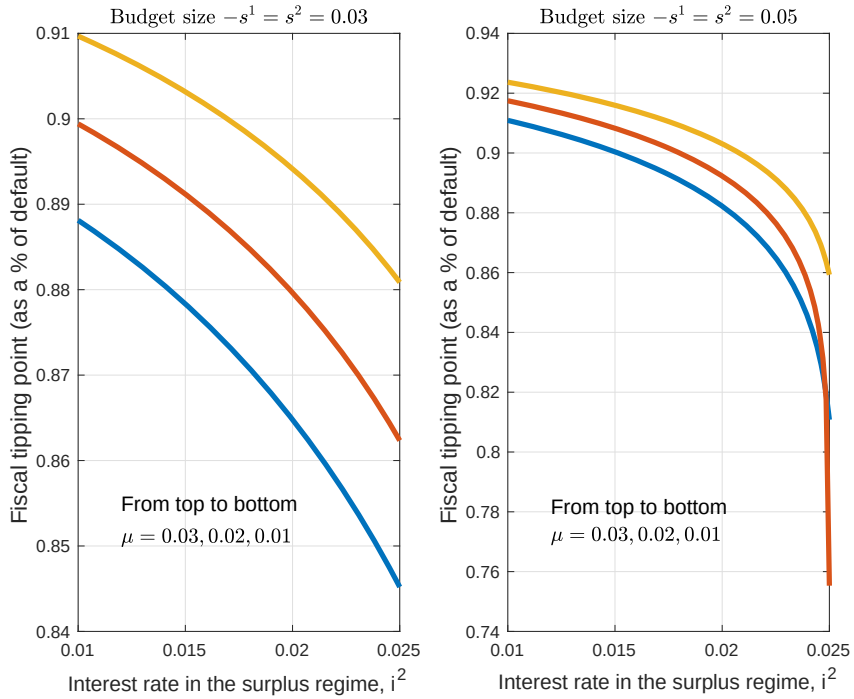
Distortionary policies I: Budget sizes and growth





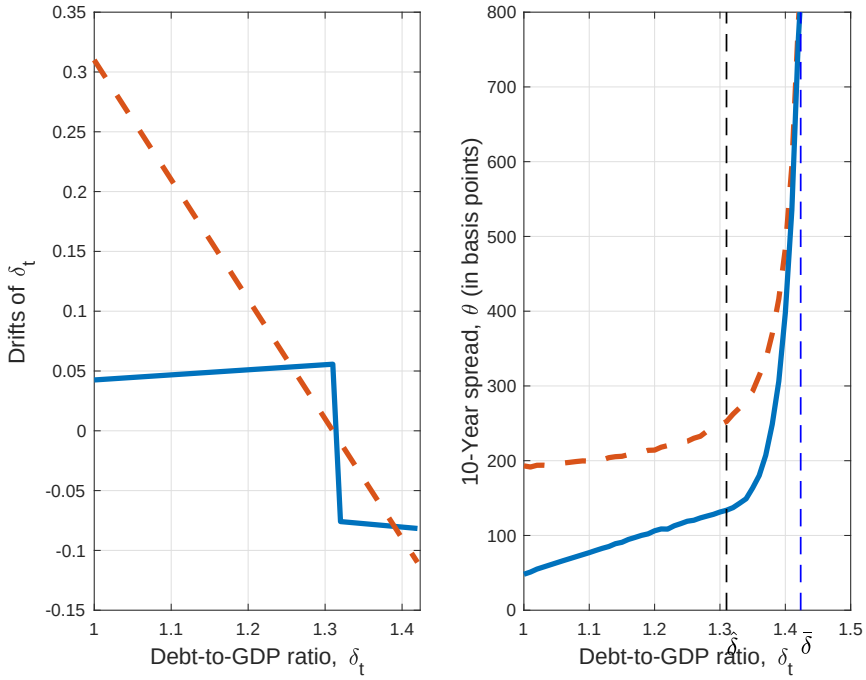
Distortionary policies II: State-dependent interest rates





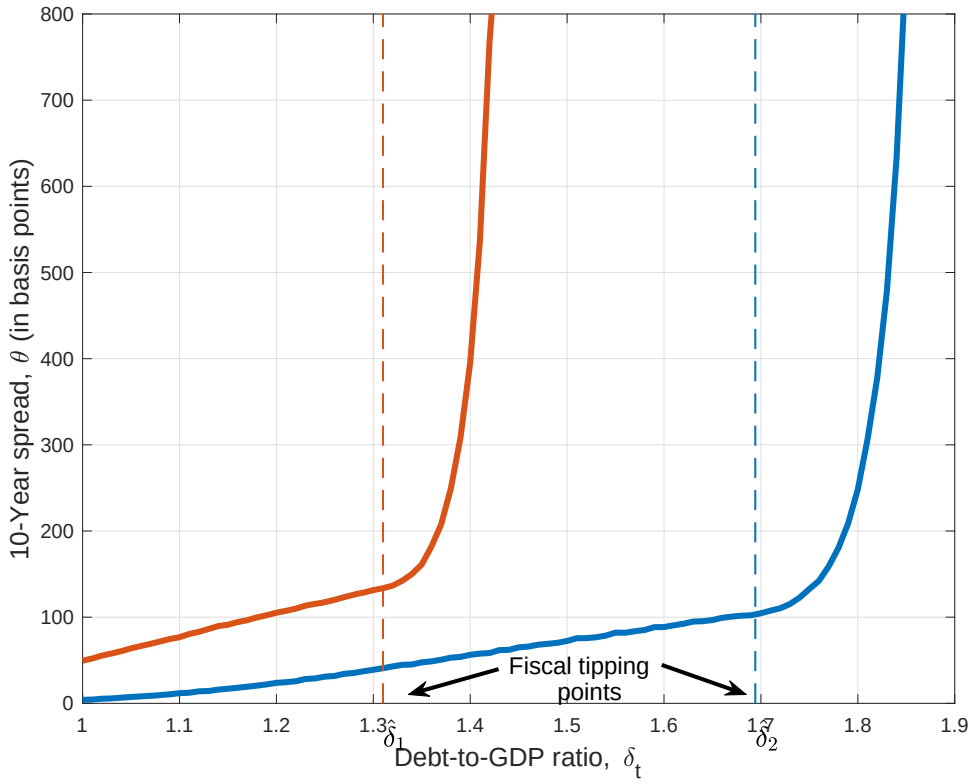
3. Spreads

Debt intolerance around fiscal tipping points

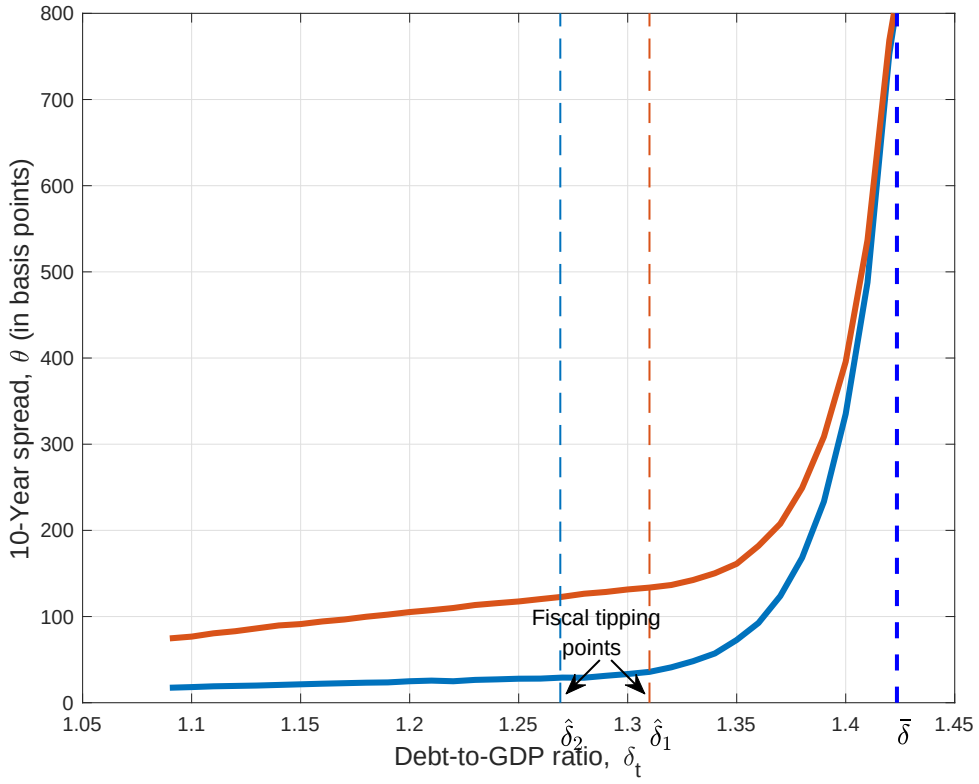


- (Non standard notion of debt intolerance.) Austerity arrives "too late," i.e., it is the very same government arguing for default probs to be unacceptably high

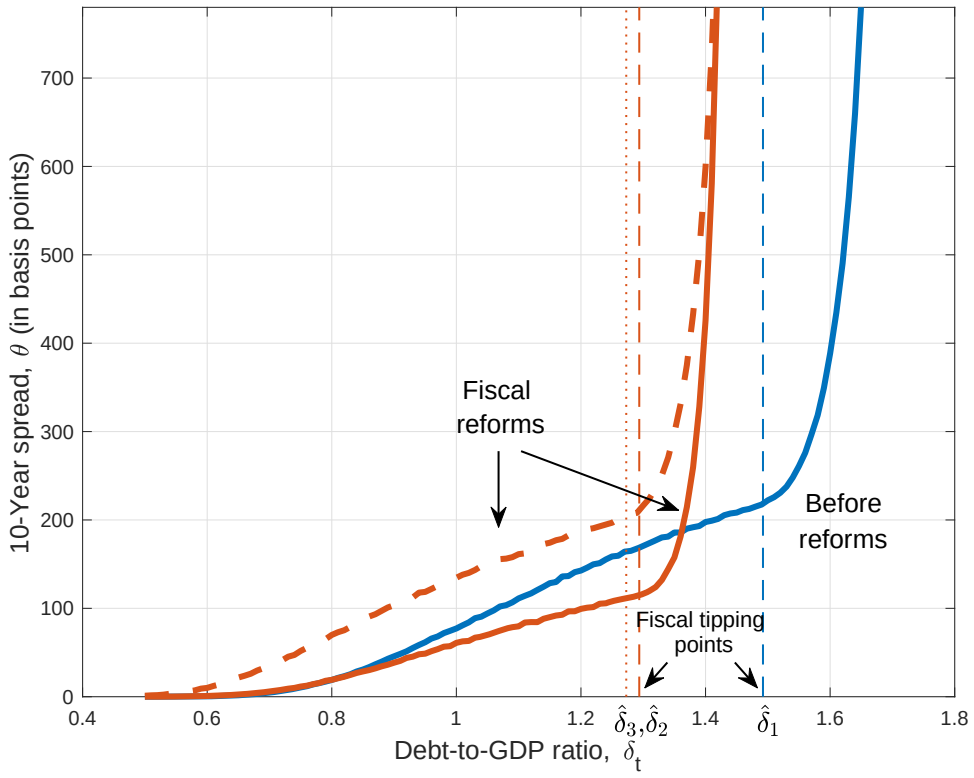
Spreads, liquidity support (e.g., QE), and moral hazard



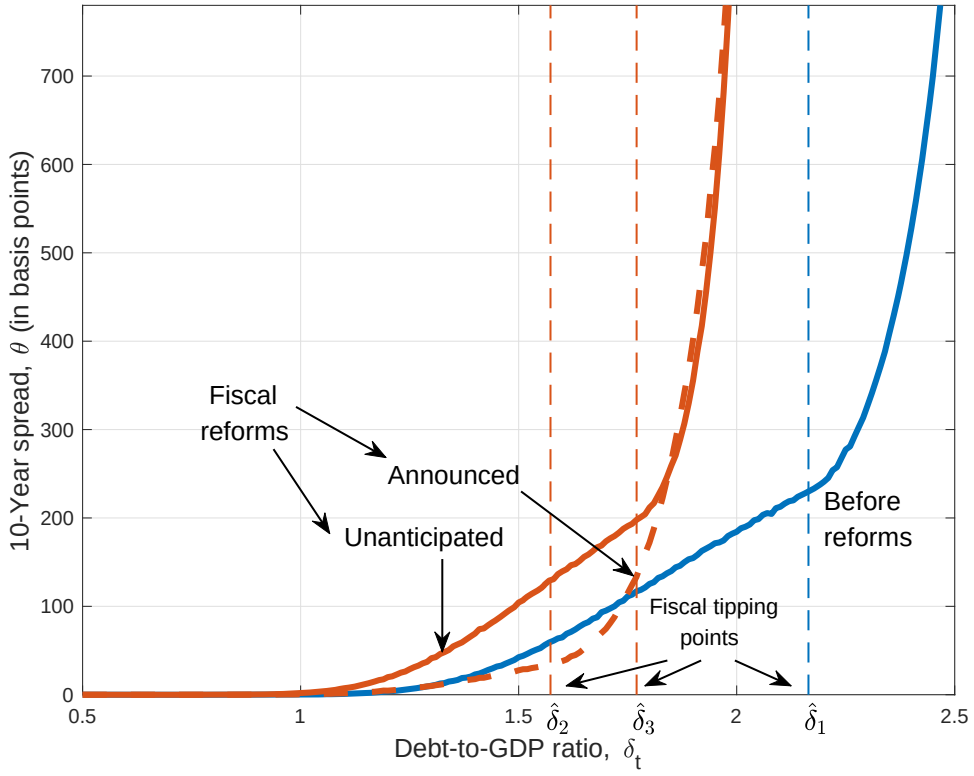
Spreads decrease with macroeconomic volatility



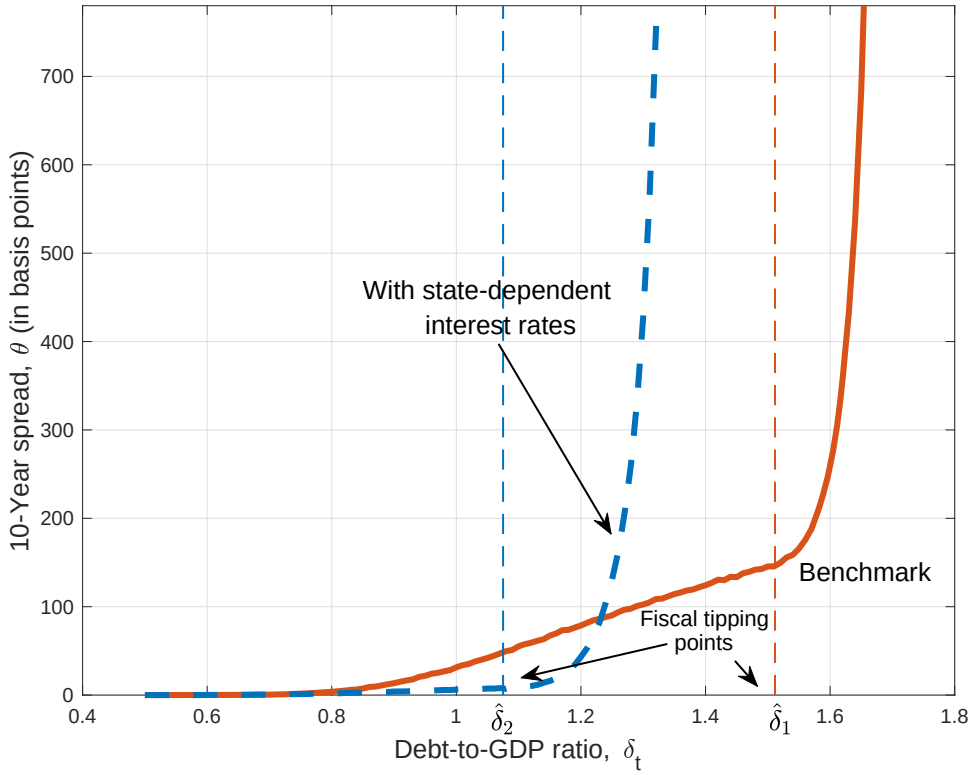
The effects of unanticipated fiscal reforms



Announced vs unanticipated fiscal reforms



State-dependent interest rates
(Or, market discipline vs moral hazard)



Conclusion

- Alesina, Favero and Giavazzi (2019) have contributed to shift Krugman v. Reinhart & Rogoff debate to
  - how to cure debt-sickness
  - provide evidence that austerity programs may be less recessive and, sometimes, even expansionary, provided these programs rely on cutting expenses rather than increasing taxes

- I ask a related question. Too much debt might lead to default, and austerity plans might be unavoidable at some point
  - $When$? Argue that plans might arrive too late to avert a debt crisis
  - Fiscal tipping points occur at 80-90% of the debt-to-GDP ratio levels that lead to default

- Testable predictions regarding how fiscal tipping points, default probabilities and premiums vary with economic growth, macroeconomic volatility, probabilities of re-entry, the size of fiscal budgets, or fiscal reforms. For example
  - spreads increase with governments' short-sightedness
  - macroeconomic stability (a public debt version of the "volatility paradox")
  - expected time defaulted governments may re-gain access to capital markets or, at least in the short-term, debt market illiquidity
  - Liquidity support (e.g., Quantitative Easing or Outright Monetary Transactions) when when debt is high
    - helps governments maintain low spreads
    - also creates moral hazard, incentivizing gvts to run deficits for longer, and increasing future debt and spreads accordingly