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Asymmetries and non-linearities in economic activity

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Industrial production is analysed for three countries. A GARCH framework is employed to model the conditional variances of the cycles, which are found to react asymmetrically to shocks of opposite sign; one of the three cases exhibits long-memory features. The ability of GARCH models at capturing all the heteroscedasticity of the data is tested against the null of deterministic chaos.

I. INTRODUCTION

Many economists, among them Keynes (1936) and Hicks (1950), suggest that business cycles are asymmetric, in that the size of the reaction of a GDP's rate of change to previously recorded shocks depends upon the sign of those shocks. Though Neftci (1984) has recently provided support for the existence of a symmetric behaviour, Beaudry and Koop (1993) as well as French and Sichel (1993) have instead evidenced asymmetry. We contribute to the debate by investigating the presence of non-linearity and asymmetry in three industrial production series, and by testing whether such features can be fully accounted for by GARCH models or if this hypothesis goes along with the presence of deterministic chaos.

II. NON-LINEARITIES AND ASYMMETRY

An increasing number of papers have recently dealt with generalized autoregressive conditionally heteroscedastic (GARCH) models, a simple and appealing scheme in which the conditional variance of a stationary series, σ_t^2, follows a restricted ARMA model, a structure that appears to capture most of time series' non-linear dynamics (Bollerslev *et al.*, 1992). However, the GARCH assumption for the variance should be tested against different forms of non-linear dependence, like deterministic chaos, whose presence has revealed hardly rejectable in many applications involving financial and macroeconomic series, based on the BDS test (Brock *et al.*, 1988).

We seek for asymmetries in the business cycle, as deduced from seasonally adjusted industrial production indices for the United States, the United Kingdom and Italy, observed monthly from January 1957 to June 1993. We use the sign-switching ARCH (SSARCH) model, a generalization of Bollerslev's (1986) GARCH developed in Fomari and Mele (1995) in which the conditional variance of a stationary but autocorrelated series, r_t, evolves according to

$$r_t = \mu + \phi\varepsilon_{t-1} + \varepsilon_t \quad \text{with } \varepsilon_t | F_{t-1} \sim N(0, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta s_{t-1} \tag{2}$$

where $\omega > 0, \alpha$ and $\beta \geqslant 0, |\omega| > |\delta|, F_{t-1}$ the information set dated $t-1$ and s_{t-1} a dummy variable, equal to 1 if r_{t-1} is positive and -1 if r_{t-1} is negative. As far as $\delta < 0$ the conditional variance reacts more strongly to past negative values of the lagged first difference of the industrial production index than it reacts to positive values.

To evidence the existence of conditional heteroscedasticity in the cycles we used Engle's (1982) TR^2, based on the regression of the squared logarithmic rates of change of the industrial production indices, $r_{i,t}$ (with $i = 1, 2, 3$), on their p previous values, which is distributed according to a chi-square with p degrees of freedom under the null of homoscedastic conditional variance; it was evaluated up to the

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Table 1. *SSGARCH models*

	ϕ	ω	α	β	δ	Pers.	Log-likelihood
USA	− 0.434	1.58E−5	0.106	0.753	− 8.93E−6	0.859	1843.2
t-statistic	− 9.53	9.52	7.20	27.54	− 3.74		
UK	0.111	1.23−4	0.373	0.163	− 3.81E−5	0.536	1673.5
t-statistic	3.78	10.31	12.21	2.83	− 4.12		
IT	0.200	4.43E−4	0.306	0.02	− 1.55E−4	0.335	1434.7
t-statistic	3.71	16.83	6.91	1.85	− 5.84		

Note: Pers. is the degree of persistence of the conditional variances, as measured by the sum $(\alpha + \beta)$.

fifth lag and equalled 114.7, 117.3 and 111.7 for Italy, the United Kingdom and the United States, respectively, highly significant at any level of confidence, which supports the presence of time-varying second moments.

A test recently developed by Engle and Ng (1993) was employed to have a preliminary idea about the presence of asymmetry. It is obtained by generating three auxiliary series: the first, z_1, is a dummy variable that equals 1 when the previous change of the industrial production index (r_{t-1}) is negative, and 0 in the remaining cases; the second, z_2, is the product of z_1 times r_{t-1}, the third, z_3, is obtained as the product of $(1 - z_1)$ by r_t. Against these series are regressed the logarithmic rates of change of the industrial production indices, and the TR^2 of such regressions (T being sample size) are distributed as a chi-square with three degrees of freedom under the null of no asymmetry. The results (commented upon but not reported) show that shocks of different sign generate different impact on the amount of volatility of the business cycle. The coefficients of the three SSGARCH models are reported in Table 1.

Asymmetry is a common feature of the series, as revealed by the significance of δ; being negative it supports the idea of negative shocks having greater impact on the variance. Our methodology employs just the sign of the last change in the production index as a conditioning variable for σ_t^2, even though the sign of the last error made in forecasting r_{t-1}, i.e. ε_{t-1}, could provide information about a general tendency of the conditional variance to behave differently following the occurrence of negative shocks. To examine such an opportunity we estimated the SSGARCH (1, 1) model incorporating both signs, i.e.

$$
\begin{gathered}
r_t = \phi\varepsilon_{t-1} + \varepsilon_t \\
\varepsilon_t | F_{t-1} \sim N(0, \sigma_t^2) \qquad (3) \\
u_t = \varepsilon_t / |\varepsilon_t| \\
s_t = r_t / |r_t| \\
\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta s_{t-1} + \gamma u_{t-1}
\end{gathered}
$$

where s_t and u_t record the sign of the last change of r_t and the sign of ε_{t-1}, respectively. In all three cases γ was found to be negative but not significant, with the *t*-statistic ranging from −1.23 (US) to −0.72 (UK), so no evidence could be found of a general tendency of the conditional variances to overreact to negative forecast errors, in addition to the finding that such behaviour follows drops in economic activity.

Diagnostic checking of the estimated models has been carried out mainly by comparing the kurtosis of the original series, standardized with their sample standard deviation in one case, and with the SSGARCH conditional standard deviation in another. The kurtoses fell from 7.27, 9.97 and 6.63 to 5.00, 5.11 and 5.00 for Italy, the United Kingdom and the United States, respectively. Such values are higher than 3, the kurtosis of a standard normal distribution, revealing limited ability of GARCH models to capture the heteroscedastic features of the data. This occurrence, instead, is not as clearly evidenced by the Pagan and Sabau (1988) test, based on the regression of the squared logarithmic rates of change of the industrial production indices on a constant and the conditional variance. The null of GARCH capturing the whole data's heteroscedasticity could not be rejected since the constant did not deviate significantly from nil and the slopes were 1.22, 1.15 and 1.43 for Italy, the United Kingdom and the United States, respectively, not significantly different from unity. Rather mixed evidence for lack of fit emerges from the two tests, so we will try to improve the current specification in the next section.

III. LONG MEMORY AND DETERMINISTIC CHAOS

To test whether it is possible to improve the results obtained from the SSGARCH, we first checked whether the variance was the optimal concept of volatility, then we sought other sources of non-linearity.

Concerning an optimal definition of volatility, recent evidence for financial variables based on the power ARCH scheme (Ding *et al*., 1993) has indicated that the variance is not the best concept of volatility. In fact, the likelihood of the power ARCH, which models the standard deviation raised to the δth power, with δ being estimated from the data, significantly exceeds the likelihood of a standard

Table 2. *Power ARCH models*

	ϕ	ω	α	β	τ	δ	Pers.	Log-likelihood
USA	− 0.377	3.17E−6	0.0026	0.562	0.512	1.13	0.565	1851.8
t-statistic	7.09	14.57	22.95	55.19	3.32	123.6		
UK	− 0.184	4.75E−11	3.3E−4	0.468	0.292	1.89	0.469	1608.8
t-statistic	4.08	15.72	38.32	19.92	4.42	557.7		
IT	0.257	0.00311	0.018	0.67	1.000	0.636	0.164	1424.0
t-statistic	8.71	62.85	62.85	45.89	Restr.	11.75		

Notes: Pers. is the degree of persistence of the conditional variances, as measured by the sum ($\alpha + \beta$).
Restr. stands for restricted.

Table 3. *SSGARCH in mean models*

	ϕ	ω	α	β	δ	τ	Pers.	Log-likelihood
USA	− 0.369	1.40E−5	0.098	0.765	− 7.1E−6	− 32.11	0.863	1851.9
t-statistic	−6.45	10.75	7.48	33.07	− 3.43	− 4.25		
UK	0.15	1.86E−4	0.368	0.211	− 2.9E−6	− 7.69	0.579	1641.9
t-statistic	2.68	10.95	11.68	4.15	− 3.22	− 2.80		
IT	0.28	3.48E−4	0.229	0.172	− 1.1E−4	− 5.99	0.401	1441.5
t-statistic	4.62	18.62	6.14	4.55	− 4.48	− 4.08		

Note: Pers. is the degree of persistence of the conditional variances, as measured by the sum ($\alpha + \beta$).

GARCH, in which δ is fixed at 2. The structure of the power ARCH is

$$r_t = \varepsilon_t + \phi\varepsilon_{t-1} \quad \text{with} \quad \varepsilon_t | F_{t-1} \sim N(0, \sigma_t^2) \tag{4}$$

$$\delta_t^{\delta} = \omega + \alpha(|\varepsilon_{t-1}| - \tau\varepsilon_{t-1})^{\delta} + \beta\sigma_{t-1}^{\delta} \tag{5}$$

with $\omega > 0, \alpha \geqslant 0, \beta \geqslant 0, -1 \leqslant \tau \leqslant 1, \delta > 0$. Estimates are reported in Table 2.

Unlike the evidence obtained for financial variables, the logarithm of the likelihood of the power ARCH for the industrial production indices is lower than the corresponding value of the SSGARCH for the United Kingdom and Italy, though significantly higher for the United States; δ equalled 0.64, 1.90 and 1.13 for Italy, the United Kingdom and the United States, respectively. Thus, modelling the conditional variance ($\delta = 2$) seems a plausible assumption only for the United Kingdom; for the remaining two countries, an asymmetric GARCH model for the standard deviation ($\delta = 1$) would be preferable.

Working with a different concept of volatility did not prove successful since non-linearity was evidenced anyway when the logarithmic rates of change of the industrial production indices were standardized with both the SSGARCH or the power ARCH standard deviation. As a viable alternative, one may investigate the presence of chaotic dynamics using a GARCH scheme, which allows estimation of a theoretical model developed by Grandmont (1985). It is based on overlapping generations composed of two representative agents, young and old. These agents live for two periods, have perfect foresight about future prices and quantities, and maximize an additive utility function of two arguments, consumption and leisure. The model generates chaotic dynamics due to time variations of the Arrow–Pratt absolute risk aversion coefficient τ, where $\tau = -(V''(\cdot))/V'(\cdot) = k$; $V'(\cdot)$ and $V''(\cdot)$ are the first and second derivatives of the indirect utility function. This scheme nests into the GARCH in mean model (Engle *et al.*, 1987) which allows for feedback between conditional mean and variance of a series and is specified as

$$r_t = \varepsilon_t + \phi\varepsilon_{t-1} + \tau\sigma_t \quad \text{with } \varepsilon_t | F_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta s_{t-1}$$

In this model τ represents the Arrow–Pratt risk aversion coefficient, measuring the curvature of the utility function. To evaluate its overall significance, we estimated the model first on the whole sample, then we applied it to rolling samples on a fixed window of 24 months, thus controlling the stability of τ over time, a necessary condition to rule out the occurrence of chaotic effects.

Table 3 reports the estimated coefficient of the SSGARCH in mean, which reveals the significance of τ, hence the presence of a relationship between conditional first and second moments of the cycles. The rolling sample estimates evidence large time variations of τ; as far as Italy is concerned it ranged between − 52 and 10; ranges were even wider for the remaining countries. Thus the amount of non-linearity that remains, even after the data have been filtered with the GARCH standard deviation, may arise from the presence of chaotic effects, originating from changes of the curvatures of the economic agents' utility function.

IV. CONCLUSIONS

We have produced novel international evidence regarding the asymmetric behaviour of the business cycles over time. Monthly conditional variances of the logarithmic rates of change of the industrial production indices appear to be higher following drops of economic activity for the United States, the United Kingdom and Italy. Following recent evidence for financial variables, we employed a different scheme which models the δth power of the conditional standard deviation, under the idea that a different concept of volatility may be more informative. However, this did not turn out to be the case. Following Grandmont (1985) a GARCH revealed the presence of chaotic dynamics, which may be an alternative explanation to the observed non-linearity.

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